Anticipated Alternative Instrument-Rate Paths in Policy Simulations

Stefan Laséen, and Lars E.O. Svensson
Sveriges Riksbank

Central Bank Macroeconomic Modeling Workshop,
Bangko Sentral ng Pilipinas, October 2010
The views, analysis, and conclusions in this paper are solely the responsibility of the authors and do not necessarily agree with those of other members of the Riksbank staff or executive board.
Introduction / Motivation

- Key issue #1: How compute a preferred path for the interest rate?
- Key issue #2: How compute alternative paths for the interest rate?

This paper is largely motivated from practical considerations

- Growing tendency among central banks to construct and report macroeconomic projections consistent with their own views about the future evolution of nominal interest rates
- Since February 2007, Riksbank conditions forecasts on a preferred path for the Repo-rate
- Transparency and ease of communication
Introduction / Motivation
What is the problem?

- Given the model
- Given historical policy behaviour

$\Rightarrow$ 1 forecast!

- Why are we interested in alternative paths?!
Introduction / Motivation
What is the problem?

- The main purpose: provide the policymaker with a set of policy choices and to illustrate how the development of the economy would differ for different policy choices.

- Historical policy behaviour might not reflect the current preferences of the executive board.

- Historical policy behaviour might not be a possible choice in the current economic situation (e.g. ZLB).

- The problem is how to do experiments with different interest rate paths which might be used as announced main scenario.
  - Efficient.
  - Respect various restrictions.
Introduction / Motivation

First:

The evolution of the repo-rate forecast and market expectations December 2008 - September 2010.
Introduction / Motivation

MPU 2008 December

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPR 2009 February

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPU 2009 April

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPR 2009 July

Anticipated Instrument-Rate Paths
Introduction / Motivation
MPU 2009 September

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPR 2009 October

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPU 2009 December

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPR 2010 February

Anticipated Instrument-Rate Paths

Percent

Year
Introduction / Motivation

MPU 2010 April

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPR 2010 July

Anticipated Instrument-Rate Paths
Introduction / Motivation

MPU 2010 September

Anticipated Instrument-Rate Paths
Introduction / Motivation

How compute a preferred path for the interest rate?
- The preferred path does e.g. not include the historically high smoothing of interest rates
- Good correspondence between CB and market expect.
Introduction / Motivation

- How compute alternative paths for the interest rate?
- Bad correspondence between CB and market expect expectations
Introduction / Motivation

- How compute alternative paths for the interest rate?
- Zero lower bound issues
Introduction / Motivation

- This paper specifies a convenient way to do policy simulations with alternative instrument-rate paths
  - The new element is that these alternative instrument-rate paths are anticipated by the private sector
  - Previous methods have instead implemented alternative instrument-rate paths by adding unanticipated shocks to an instrument rule
- Many central banks are familiar with the concept of anticipated shocks
  - See for instance Harrison et al. (2005) for an application of this algorithm.
Introduction / Motivation

- Is the equilibrium unique?
- In our case it is
  - Interest rate path exogeneous for $T$ periods only
  - Return to rule or optimal policy in period $T + 1$
Some illustrative results/observations

1. Policy advice on preferred path and alternatives becomes tricky when the path is announced
   - Can, or should, the alternative paths be seen as possible main scenarios?
   - Reason for the alternative path?
2. The whole path matters!
   - Is the interest rate path credible? "Forecast, not a promise"
3. Focus on the real interest rate!
   - How are expectations formed?
Some illustrative results/observations
1. Tricky policy advice - expectations and communication/credibility
Some illustrative results/observations
2. Adding one quarter of low expected interest rate makes a difference!?
Some illustrative results/observations

3. Different restrictions but same real interest rate

- Expected real interest rate
- Unexpected nominal interest rate

- Anticipated Instrument-Rate Paths
The method / theory

The model can be written:

- **State space form:**

  \[
  \begin{bmatrix}
  X_{t+1} \\
  Hx_{t+1|t}
  \end{bmatrix} = A \begin{bmatrix}
  X_t \\
  x_t
  \end{bmatrix} + Bi_t + \begin{bmatrix}
  C \\
  0
  \end{bmatrix} \varepsilon_{t+1}
  \]

- \(X_t\) predetermined variables in quarter \(t\), \(x_t\) forward-looking variables, \(i_t\) instrument rate, \(\varepsilon_{t+1}\) i.i.d. shock, \(E_t \varepsilon_{t+1} = 0, E_t \varepsilon_{t+1} \varepsilon'_{t+1} = I_{n_\varepsilon} x_{t+1|t} \equiv E_t x_{t+1}\)

- Policy is given by a general policy rule

  \[G_x x_{t+1,t} + G_i i_{t+1,t} = f_X X_t + f_x x_t + f_i i_t.\]
Let $u^t \equiv \{u_{t+\tau}, t\}_{\tau=0}^{\infty}$ denote a projection in period $t$ for any vector of variables $u_t$. The projection model for the projections $(X^t, x^t, i^t)$ in period $t$ uses that the projection of the zero-mean i.i.d. shocks is zero, $\varepsilon_{t+\tau} = 0$ for $\tau \geq 1$. It can then be written as

$$
\begin{bmatrix}
X_{t+\tau+1} \\
Hx_{t+\tau+1}
\end{bmatrix}
= A
\begin{bmatrix}
X_{t+\tau} \\
x_{t+\tau}
\end{bmatrix}
+ Bi_{t+\tau},
$$

for $\tau \geq 0$, where

$$
X_{t,t} = X_{t|t},
$$

where $X_{t|t}$ is the estimate of predetermined variables in period $t$ conditional on information available in the beginning of period $t$. The feasible set of projections for given $X_{t|t}$, denoted $\mathcal{T}(X_{t|t})$, is the set of projections that satisfy (3)-(4).
The method / theory

Issue 1: The preferred path / optimal policy choice

The optimal projection is the projection \((\hat{X}_t, \hat{x}_t, \hat{i}_t, \hat{Y}_t)\) that minimizes the intertemporal loss function,

\[
L_t(Y_t) = \sum_{\tau=0}^{\infty} \delta^\tau L_{t+\tau,t}, \text{ where } L_{t+\tau,t} = Y_{t+\tau,t}' W Y_{t+\tau,t} \quad (5)
\]

for \(\tau \geq 0\). \(Y_t\) is an \(n_Y\)-vector of target variables, measured as the difference from an \(n_Y\)-vector \(Y^*\) of target levels.

The optimal policy choice, which results in the optimal policy projection, can now be formulized as

\[
\min L_t(Y_t) + \frac{1}{\delta} \Xi_t' H(x_{t,t} - x_{t,t-1}) \text{ subject to } (X_t, x_t, i_t, Y_t) \in \mathcal{T}_t(X_t|t).
\]
The method / theory

Issue 1: The preferred path / optimal policy choice

\[ \mathcal{T}(X_{t|t}) \]

\[ (\hat{\pi}^t, \hat{y}^t - \bar{y}^t) \]

\[ \mathcal{L}(Y^t) + \frac{1}{\delta} \Xi'_{t-1} H(x_{t,t} - x_{t,t-1}) = \text{const.} \]
The method / theory
Issue 1: The preferred path / optimal policy choice

Under the assumption of optimization under commitment in a timeless perspective, the optimal projection can be described by the following difference equation,

\[
\begin{bmatrix}
\hat{x}_{t+\tau,t} \\
\hat{i}_{t+\tau,t}
\end{bmatrix}
= \begin{bmatrix}
F_x \\
F_i
\end{bmatrix}
\begin{bmatrix}
\hat{X}_{t+\tau,t} \\
\Xi_{t+\tau-1,t}
\end{bmatrix}, \quad \begin{bmatrix}
\hat{X}_{t+\tau+1,t} \\
\Xi_{t+\tau,t}
\end{bmatrix}
= M \begin{bmatrix}
\hat{X}_{t+\tau,t} \\
\Xi_{t+\tau-1,t}
\end{bmatrix}
\tag{6}
\]

\[
\hat{Y}_{t+\tau,t} = D \begin{bmatrix}
\hat{X}_{t+\tau,t} \\
\hat{x}_{t+\tau,t} \\
\hat{i}_{t+\tau,t}
\end{bmatrix},
\tag{7}
\]

for \( \tau \geq 0 \), where \( \hat{X}_{t,t} = X_{t|t} \). The submatrix \( F_i \) in (6) represents the optimal instrument rule,

\[
i_{t,t+\tau} = F_i \begin{bmatrix}
X_{t+\tau,t} \\
\Xi_{t+\tau-1,t}
\end{bmatrix}.
\tag{8}\]
By varying the parameters of the modified intertemporal loss function it is possible to generate alternative policy projections.

The policymaker may still prefer to see a few representative alternative policy projections constructed with alternative instrument-rate paths that are not constructed as optimal policy projections. The methods to construct policy projections for alternative anticipated instrument-rate paths presented in this paper makes that possible.
Consider a restriction on the instrument-rate projection of the form
\[ i_{t+\tau,t} = \bar{i}_{t+\tau,t}, \quad \tau = 0, \ldots, T, \] (9)
where \( \{\bar{i}_{t+\tau,t}\}_{\tau=0}^{T} \) is a sequence of \( T + 1 \) given instrument-rate levels. Alternatively, we can have restriction on the real (CPI) instrument-rate projection of the form
\[ r_{t+\tau,t} = \bar{r}_{t+\tau,t}, \quad \tau = 0, \ldots, T. \] (10)
The method / theory

Issue 2: The restrictions / alternative anticipated paths

The restrictions can be implemented by:

1. Adding time-varying constants to the policy rule (Svensson 2005, IJCB),

   \[ G_{x} x_{t+\tau+1,t} + G_{i} i_{t+\tau+1,t} = f_{x} X_{t+\tau,t} + f_{x} x_{t+\tau,t} + f_{i} i_{t+\tau,t} + z_{t+\tau,t}, \]

   \[ \tau = 0, \ldots, T, \]

   where the sequence of scalars \( \{z_{t+\tau,t}\}_{\tau=0}^{T} \) is chosen such that (9) or (10) is satisfied.

2. We also work with:

   a. A time-varying general policy rule

      \[ G_{xt} x_{t+\tau+1,t} + G_{it} i_{t+\tau+1,t} = f_{xt} X_{t+\tau,t} + f_{xt} x_{t+\tau,t} + f_{it} i_{t+\tau,t}, \]

   b. With linear equality-constraints in a finite-horizon projection model
The method / theory

Issue 2: The restrictions / alternative anticipated paths

The restrictions can be implemented by:

1. **Adding time-varying constants to the policy rule** (Svensson 2005, IJCB),

   \[ G_{x_t + \tau + 1, t} + G_{i_t + \tau + 1, t} = f_{X_t + \tau, t} + f_{x_t + \tau, t} + f_{i_t + \tau, t} + z_{t + \tau}, \]

   \( \tau = 0, ..., T, \) where the sequence of scalars \( \{z_{t + \tau, t}\}_{\tau=0}^T \) is chosen such that (9) or (10) is satisfied.

2. **We also work with:**

   1. **A time-varying general policy rule**

      \[ G_{x_t + \tau + 1, t} + G_{i_t + \tau + 1, t} = f_{X_t + \tau, t} + f_{x_t + \tau, t} + f_{i_t + \tau, t}, \]

   2. **With linear equality-constraints in a finite-horizon projection model**
The method / theory

Issue 2: The restrictions / alternative anticipated paths

The restrictions can be implemented by:

1. Adding time-varying constants to the policy rule (Svensson 2005, IJCB),

\[
G_{\tau}x_{t+\tau+1,t} + G_{\tau}i_{t+\tau+1,t} = f_{\tau}X_{t+\tau,t} + f_{\tau}x_{t+\tau,t} + f_{\tau}i_{t+\tau,t} + z_{t+\tau,t},
\]

(11)

\[\tau = 0, \ldots, T,\] where the sequence of scalars \{z_{t+\tau,t}\}_{\tau=0}^{T} is chosen such that (9) or (10) is satisfied.

2. We also work with:

1. A time-varying general policy rule

\[
G_{\tau}x_{t+\tau+1,t} + G_{\tau}i_{t+\tau+1,t} = f_{\tau}X_{t+\tau,t} + f_{\tau}x_{t+\tau,t} + f_{\tau}i_{t+\tau,t},
\]

(12)

2. With linear equality-constraints in a finite-horizon projection model
The method / theory

Issue 2: The restrictions / alternative anticipated paths

The restrictions can be implemented by:

1. Adding time-varying constants to the policy rule (Svensson 2005, IJCB),

\[
G_x x_{t+\tau+1,t} + G_i i_{t+\tau+1,t} = f_X X_{t+\tau,t} + f_x x_{t+\tau,t} + f_i i_{t+\tau,t} + z_{t+\tau,t},
\]

(11)

\(\tau = 0, ..., T\), where the sequence of scalars \(\{z_{t+\tau,t}\}_{\tau=0}^T\) is chosen such that (9) or (10) is satisfied.

2. We also work with:

   1. A time-varying general policy rule

\[
G_x x_{t+\tau+1,t} + G_i i_{t+\tau+1,t} = f_X X_{t+\tau,t} + f_x x_{t+\tau,t} + f_i i_{t+\tau,t},
\]

(12)

   2. With linear equality-constraints in a finite-horizon projection model
Example

1. Forward-looking model
2. Backward-looking model
3. Ramses Model (Riksbank DSGE Model)
Example
Lindé Model

\[ \pi_t = 0.457 \pi_{t+1|t} + (1 - 0.457) \pi_{t-1} + 0.048 y_t + \epsilon_{\pi t}, \]
\[ y_t = 0.425 y_{t+1|t} + (1 - 0.425) y_{t-1} - 0.156 (i_t - \pi_{t+1|t}) + \epsilon_{yt}. \]

\[ L_t = \frac{1}{2} [\pi_t^2 + y_t^2 + 0.2 (i_t - i_{t-1})^2] \]
\[ i_t = 1.5 \pi_t + 0.5 y_t. \]

The period is a quarter, and \( \pi_t \) is quarterly GDP inflation measured in percentage points at an annual rate, \( y_t \) is the output gap measured in percentage points, and \( i_t \) is the quarterly average of the federal-funds rate, measured in percentage points at an annual rate.
Example
Rudebusch-Svensson Model

\[ \pi_t = 0.7\pi_{t-1} - 0.1\pi_{t-2} + 0.28\pi_{t-3} + 0.12\pi_{t-4} + 0.14y_t + \varepsilon_{\pi t}, \]

\[ y_t = 1.16y_{t-1} - 0.25y_{t-2} - 0.10 \left( \frac{1}{4} \sum_{j=0}^{3} i_{t-j} - \frac{1}{4} \sum_{j=0}^{3} \pi_{t-j} \right) + \varepsilon_{y t}. \]

\[ L_t = \frac{1}{2} \left[ \pi_t^2 + y_t^2 + 0.2(i_t - i_{t-1})^2 \right] \]

\[ i_t = 1.5\pi_t + 0.5y_t. \]
Example
Ramses Model (Riksbank DSGE Model)

\[
\begin{bmatrix}
X_{t+1} \\
Hx_{t+1|t}
\end{bmatrix} = A \begin{bmatrix}
X_t \\
x_t
\end{bmatrix} + B i_t + \begin{bmatrix}
C \\
0
\end{bmatrix} \varepsilon_{t+1}.
\]

\[
i_t = \rho_R i_{t-1} + (1 - \rho_R) \left( \hat{\pi}^c_t + r_\pi \left( \hat{\pi}^c_{t-1} - \hat{\pi}^c_t \right) + r_y \hat{y}_{t-1} + r_x \hat{x}_{t-1} \right) + r_{\Delta \pi} \Delta \hat{\pi}^c_t + r_{\Delta y} \Delta \hat{y}_t + \varepsilon_{R,t}
\]
Example
What we do.

1. No restrictions on $r_{t+\tau}$ or $i_{t+\tau}$ (column 1)
2. Calculate restrictions on the $r_{t+\tau}$ (column 2)
3. Calculate restrictions on the $i_{t+\tau}$ (column 3)

- Policy behaviour: Do this for different policy rules
  - Optimal (row 1)
  - Taylor (row 2)

- Horizon: Let $\tau = 0, \ldots, 5$ (slides)
- Expectations: Do this for the 3 different models (slides)
Results
Lindé model, 4 quarter restriction,

\[ \bar{t}_{t+\tau} = \bar{r}_{t+\tau} = 1, \ \tau = 0, \ldots, 3 \]

Optimal policy (1st row) and Taylor rule (2nd row)
Results
Lindé model, 5 quarter restriction,

\[ \bar{I}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 4 \]

Optimal policy (1st row) and Taylor rule (2nd row)
Results
Lindé model, 6 quarter restriction,

\[ \bar{t}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 5 \]

Optimal policy (1st row) and Taylor rule (2nd row)
Results
Unusual equilibria

- If inflation is sufficiently sensitive to the real instrument rate, “unusual” equilibria may result from restrictions on the nominal instrument rate.

- A shift up of the real interest-rate path reduces inflation and inflation-expectations so much that the nominal interest-rate path \( i_t = r_t + \pi_t^e \) shifts down.

- Then, a shift up of the nominal interest-rate path requires an equilibrium where the path of inflation and inflation expectations shifts up more and the real instrument-rate path shifts down.
Results

Rudebusch-Svensson Model, 4 quarter restriction,

$$\bar{t}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 3$$

Optimal policy (1st row) and Taylor rule (2nd row)
Results

Rudebusch-Svensson Model, 5 quarter restriction,

\[ \bar{t}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 4 \]

Optimal policy (1st row) and Taylor rule (2nd row)
Results
Rudebusch-Svensson Model, 6 quarter restriction,

\[ \bar{t}_{t+\tau} = \bar{r}_{t+\tau} = 1, \ \tau = 0, \ldots, 5 \]

Optimal policy (1st row) and Taylor rule (2nd row)
Results
Ramses Model, 4 quarter restriction,

\[ \tilde{r}_{t+\tau} = \tilde{r}_{t+\tau} = 1, \tau = 0, \ldots, 3 \]
Results
Ramses Model, 5 quarter restriction,

\[ \bar{r}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 4 \]
Results
Ramses Model, 6 quarter restriction,

\[ \bar{\pi}_{t+\tau} = \bar{r}_{t+\tau} = 1, \tau = 0, \ldots, 5 \]
Conclusions

- This paper specifies a convenient and simple way to do policy simulations with alternative instrument-rate paths
  - The new element is that these alternative instrument-rate paths are anticipated by the private sector
  - Previous methods have instead implemented alternative instrument-rate paths by adding unanticipated shocks to an instrument rule
  - Deviations are in practice both serially correlated and large, which seems inconsistent with the assumption that they remain unanticipated
Conclusions

- Restrictions on *nominal* interest rate for several quarters can be problematic when inflation is sensitive to the real rate.
- Restrictions on *real* interest rate are less problematic.
- The time horizon of the restriction shows that serially correlated shocks affect expectations.
- The results depend very much on the policy rule in place.
- The systematic part of policy eventually kicks in and will in equilibrium dominate the unsystematic part of policy.
- This depends on the expectation formation:
  - Forward-looking model: large effects
  - Backward-looking models: small effects
The restrictions can be implemented by:

- Adding time-varying constants to the policy rule (Svensson 2005, IJCB),

\[
G_{x,t} + G_{i,t} = f_X X_{t+\tau} + f_x x_{t+\tau} + f_{i,t} + z_{t+\tau},
\]

(13)

\[\tau = 0, \ldots, T,\] where the sequence of scalars \(\{z_{t+\tau}, t\}^{T}_{\tau=0}\) is chosen such that (9) or (10) is satisfied.
We assume that the deviation satisfies

$$z_t = \eta_{t,t} + \sum_{s=1}^{T} \eta_{t,t-s}.$$  

The dynamics of the deviation can be written

$$z^{t+1} = A_z z^t + \eta^{t+1}, \quad (14)$$

where the $(T + 1) \times (T + 1)$ matrix $A_z$ is defined as

$$A_z \equiv \begin{bmatrix} 0_{T\times1} & I_{T} \\ 0 & 0_{1\times T} \end{bmatrix}.$$  

Hence, $z^t$ is the central bank’s mean projection of current and future deviations, and $\eta^t$ can be interpreted as the new information the central bank receives in the beginning of period $t$ about those deviations.

Combine the projection model, (3), with the restriction (13) and the dynamics of the deviation, (14).
We can then write the combined projection model as

\[
\begin{bmatrix}
\tilde{X}_{t+\tau+1,t} \\
\tilde{H}\tilde{x}_{t+\tau+1,t}
\end{bmatrix} = \tilde{A}
\begin{bmatrix}
\tilde{X}_{t+\tau,t} \\
\tilde{x}_{t+\tau,t}
\end{bmatrix}
\]  

(15)

for \(\tau \geq 0\), where

\[
\tilde{X}_t \equiv \begin{bmatrix} X_t \\ z_t \end{bmatrix}, \quad \tilde{x}_t \equiv \begin{bmatrix} x_t \\ i_t \end{bmatrix}, \quad \tilde{H} \equiv \begin{bmatrix} H & 0 \\ G_x & G_i \end{bmatrix}.
\]
Under the assumption of the saddlepoint property, the system of difference equations (15) has a unique solution and there exist unique matrices $M$ and $F$ such that the solution can be written

\[
\tilde{X}_{t+\tau,t} = M^\tau \tilde{X}_{t,t},
\]
\[
\tilde{x}_{t+\tau,t} = F \tilde{X}_{t+\tau,t} = FM^\tau \tilde{X}_{t,t}
\]

for $\tau \geq 0$, where $X_{t,t}$ in $\tilde{X}_{t,t} \equiv (X'_{t,t}, z'^t)'$ is given but the $(T + 1)$-vector $z^t$ remains to be determined. Its elements are then determined by the restrictions (9) or (10).

In order to satisfy the restriction (9) on the nominal instrument rate, we note that it can now be written

\[
i_{t+\tau,t} = F_i M^\tau \begin{bmatrix} X_{t,t} \\ z^t \end{bmatrix} = \bar{i}_{t+\tau,t}, \quad \tau = 0, 1, ..., T.
\]

This provides $T + 1$ linear equations for the $T + 1$ elements of $z^t$. 