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***ASYMMETRIC EFFECTS OF MONETARY POLICY IN DIFFERENT PHASES OF  
ARMENIA'S BUSINESS CYCLE***

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August 2018

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**Abstract**

This paper develops empirical models and shows the presence of asymmetric responses of inflation and output in Armenia to the same size of positive and negative monetary policy shocks. Tight monetary policy yields more reduction in output compared to the increase of output in a response to the same size of loose monetary policy. On the other hand, relatively more inflation is created by expansionary policy. The theoretical micro founded model with New Keynesian frictions is developed to explain asymmetries in transmission mechanism of policy. The model is estimated for the Armenian economy using fifteen macroeconomic time series and fifteen structural shocks. Impulse response functions of second order approximated theoretical model, based on estimated structural parameters, match asymmetries from empirical models. The methodology of mixed equations is applied to calculate the contribution of the particular friction in a creation of asymmetry in the transmission mechanism. The asymmetric response of inflation is mostly the result of highly convex Phillips curve of importers. Another part of asymmetry in inflation is created by internal economy's price setting frictions and labor market rigidities. The significant part of asymmetric response in output is created by nonlinearities in capital and labor markets. Adding curvatures of the small open economy into the second order approximated model, the size of asymmetry increases through the channel of the high asymmetric reaction of real exchange rate. Third order theoretical moments of simulated models match directions and sizes of observed data. Variance decomposition of output shows that both demand and supply shocks are important drivers of output. The paper does policy experiments in demand and supply driven business cycle environments. In a demand driven growing economy, the aggressive contractionary monetary policy accelerates the decline of output with diminishing effect on inflation. Aggressive expansionary monetary policy increases the efficiency of creating inflation and decreases the stimulation of output in a demand driven recession. When the economy is in supply driven expansion, the increase in reaction of monetary policy accelerates the decline in output with no significant relative impact on inflation. In a supply driven recession, the aggressive response increases the reaction of output with diminishing effect on inflation.

**JEL classification:** C32, E12, E32, E52

**Keywords:** Nonlinear VAR, Simulation, New Keynesian DSGE, Monetary Policy, Asymmetries, Business Cycle, Expansion, Recession, Asymmetric Effects of Monetary Policy

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\* The views expressed in this paper are those of the author and do not necessarily represent the views or policies of the Central Bank of Armenia

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## 1. Introduction

Whether the tight and easy monetary policies have asymmetric effects on inflation and output must be in attention of policy makers. The size of asymmetry shows the efficiency of expansionary and contractionary policies. Another important policy issue is the power of central banks to stimulate the economy during recessions and prevent the growing economy from overheating. Additionally, the efficiency of monetary policy at the particular point of business cycle, driven either by demand or supply shocks, might be different. The first purpose of this paper is to show whether monetary policy has asymmetric effects on the Armenian economy at the steady state. The second purpose is to find the main sources of these asymmetries. The final purpose of the paper is to estimate efficiencies of modest and aggressive policies in recessions and expansions, and formulate policy suggestions.

Literature presents the evidence of the asymmetric response of the economy to interest rate. At the first stage, this paper does two stage estimation and finds that the sum of short and long-term impacts of contractionary policy reduces inflation by 0.62% in Armenia. The estimated impact on output is -0.91%. The net effect of monetary policy reduces output by 0.72% and increases inflation by 0.22%. Then the nonlinear VAR model is constructed to present the dynamic responses of output and inflation to positive and negative monetary policy shocks. The empirical models' results show that 1% of tight policy reduces output more compared to the increase of output in a response to the same size of loose policy.

Reduced form empirical models are not useful to explain the sources of asymmetries. So, paper develops New Keynesian small open economy DSGE model with nonlinear frictions to explain the reasons of asymmetries in the transmission mechanism theoretically. The model has nonlinear price setting frictions in domestic good production sector, export and import sectors. Rigidities in labor market are introduced via Calvo type wage mechanism. The paper discusses nonlinear investment adjustment costs in capital market, which is not visible in the linearized model and has effect on model's dynamics in higher order approximations.

The model is estimated for the Armenian economy using Bayesian estimation method. Fifteen macroeconomic variables and fifteen structural shocks are included in the estimation process. The posterior estimation results report 0.9 price stickiness coefficient of domestically

produced goods. The estimated stickiness parameter of imported consumption goods is 0.6. Higher price adjustment frequency of imported consumption good's sector makes the Phillips curve more convex. Investment adjustment costs parameter is estimated to 5.25, which is close to values widely used in business cycle literature. The posterior estimated elasticity of substitution between varieties of labor is higher than the prior belief. The estimated value is 7, which increases curvatures of the labor market block. But the estimated 0.84 stickiness parameter of wages reduces the wage Phillips curve's convexity.

The widely used approach to show asymmetries in DSGE models is the application of second order perturbation method to the nonlinear model. The disadvantage of fully second order approximated model is its disability to estimate the contribution of the specific friction in a creation of asymmetries in transmission mechanism of monetary policy. The novelty of this paper is that it develops and uses the methodology of mixed equations. The methodology enables to estimate the role of the particular sector and friction in the creation of asymmetry.

Impulse responses of the theoretical model reproduce the dynamics of asymmetries generated by the empirical model. In addition, empirical third order moments of the model's data generation process reproduces the business cycle asymmetry observed in Armenia. The nonlinear standard parts of the model together with log-linearized rigidities do not have strong curvatures to reproduce asymmetries. Paper finds that the asymmetry in inflation is mostly created by the price setting frictions of importers. The asymmetry in output arises when having either nonlinear capital market frictions or labor market frictions. The model with linearized external sector and nonlinear internal economy explains much of the asymmetry in output. The external sector's curvatures contribute to the asymmetry of real sector through the channel of the import and the increase in asymmetry of real exchange rate.

The variance decomposition of inflation and output reports the importance of demand and supply side shocks. The paper then does policy experiments in different phases of business cycle when it is driven either by demand or supply side shocks. The growing economy is defined as the output, which is 5% above steady state. The recession is the output, which is 5% below from steady state. Comparing the same size of contractionary and expansionary monetary policies in demand driven growing economy and recession, the tight policy is more effective to

control inflation and less effective to create inflation in recession. The efficiency of aggressive monetary policy to prevent inflation is decreasing because the trade-off between inflation and output is increasing when the economy is going to the direction of steady state. On the other hand, the efficiency of aggressive policy on inflation in demand driven recession is increasing because demand driven recession moves aggregate demand to a flatter region of the economy's aggregate supply. Results of the policy experiment report that monetary policy has relatively higher impact on inflation in supply driven recessions rather than expansions. On the other hand, the efficiency of aggressive monetary policy on inflation in supply driven recession is decreasing because the supply driven recession intersects the aggregate demand curve at its steeper part.

The reminder of the paper is organized as follow. Section 2 finds the presence of asymmetric effects of monetary policy on inflation and output in Armenia. Section 3 develops the micro founded theoretical model. Section 4 describes data and estimation process of the model. Section 5 discusses the sources of curvatures in the model. Section 6 presents the second order perturbation method. Section 7 discusses the mixed equations approach. Section 8 discusses asymmetries in the transmission mechanism of monetary policy in Armenia. Section 9 tests the ability of the model to reproduce business cycle asymmetries observed in Armenia. Section 10 estimates the efficiency of monetary policy in demand and supply driven recessions and expansions. Finally, Section 11 concludes.

## **2. Empirical Evidence of Asymmetric Effects of Monetary Policy in Armenia**

The asymmetric effect of monetary policy has been in attention of economists during past three decades. DeLong and Summers (1988) show that expansionary monetary policy has the greater effect on U.S. output than the tight policy. Cover (1992), using U.S. postwar data, finds that the increase in money supply is less effective to stimulate output compared to the effect of decrease in money supply. Morgan (1993), using federal funds rate as the policy instrument, tests the same hypothesis and gets the similar results. Ravn and Sola (1997) find no asymmetric impact between positive and negative monetary policy shocks. Instead, authors find the asymmetry between relative impacts of small and large shocks. Karras (1996) shows the evidence of

asymmetry of money supply on output for 18 European countries. Using the methodology of rolling VARs, Wong (2000) finds the relatively higher responses of U.S. output and price level to positive monetary policy shock. Garcia and Schaller (2002) test the impact of policy on output during recessions and expansions. They use Markov switching model and find, that monetary policy is more effective in recessions. Peersman and Smets (2001) use a multivariate extension of Hamilton's two-state Markov Switching Model and show that monetary policy is more effective in recessions in Euro area. Kaufmann (2002) shows the relative efficiency of monetary policy in recessions using Austrian data. Weise C. L. (1999) shows the asymmetric reaction of U.S. economy to monetary policy employing LSTVAR model. Lo and Piger (2005) provide the strong evidence of asymmetry in business cycle by estimating an unobserved components model. Fehr and Tyran (2001) present the evidence of asymmetric effects of expansionary and contractionary monetary policies as a result of money illusion.

In summary, literature presents empirical evidence of asymmetric effects of monetary policy. The rest of this section empirically tests the presence of such types of asymmetries in Armenia.

### **2.1. *Asymmetries in the Main Macroeconomic Variables of Armenia***

Models commonly used in macroeconomic analysis are linear models, which generate symmetric time series. The simplest way to see asymmetries in macroeconomic time series is the calculation of skewness. The asymmetric statistics of interbank interest rate, seasonally adjusted quarterly percentage changes in consumer price index and GDP growth in Armenia are presented in Table 1. The sample starts from the first quarter of 2004 and ends in the first quarter of 2017. According to the results, inflation and interest rate have a positive skewness. On the other hand, the distribution of economic growth is negatively skewed.

Table 1. Skewness

Q/Q Inflation	0.16
Q/Q GDP Growth	-1.23
Interest Rate	0.42

Log-linearized models, commonly used in macroeconomic policy analysis institutions, generate time series with zero skewness. The asymmetries are the result of higher order terms,

such as price dispersion, investment adjustment costs, wage dispersion, capital utilization, which are not visible in the first order.

## 2.2. Two Stage Estimation

This part of the paper shows the presence of asymmetric effects of monetary policy on the main macroeconomic variables in Armenia using the methodology proposed in Cover (1992). The logic of methodology is the following. First stage estimates the interest rate rule to get residuals of the equation. At the second stage, inflation and economic growth equations are estimated as functions of policy stances.

The sample period for the estimation is from the first quarter of 2004 to the first quarter of 2017. As an instrument of monetary policy, interbank interest rate is used. The other two variables are consumer price index and GDP, which are seasonally adjusted using X12 algorithm. Then, the quarterly percentage changes are calculated to get inflation and economic growth. The following interest rate equation is estimated.

$$R_t = Constant + \rho_R R_{t-1} + \mu_\pi \pi_t + \varepsilon_t \quad (2.1)$$

Table 2 shows the estimation results of the simple interest rate rule. The results are statistically significant. Interest rate is mostly driven by its previous lag. 1% inflation causes 0.21% hike in interest rate.

Table 2. OLS Estimation results of the interest rate equation

$\rho_R$	<b>0.905***</b> (0.061)
$\mu_\pi$	0.211* (0.107)
Constant	0.014 (0.153)
$R^2$	0.815

Note. Standard errors are in parentheses.

\* and \*\*\* significant at 0.1 and 0.01 respectively

Next step estimates responses of economic growth and inflation to easy and tight monetary policy. Residuals of the estimated interest rate equation represent the stance of monetary policy. The following two equations are estimated for Armenia:



$$\Delta y_t = a_0 + a_1 \Delta y_{t-1} + \sum_{i=1}^4 a_{2,i} Policy_{t-i}^+ + \sum_{i=1}^4 a_{3,i} Policy_{t-i}^- + \varepsilon_t^y \quad (2.1)$$

$$\pi_t = b_0 + b_1 \pi_{t-1} + \sum_{i=1}^4 b_{2,i} Policy_{t-i}^+ + \sum_{i=1}^4 b_{3,i} Policy_{t-i}^- + \varepsilon_t^\pi \quad (2.2)$$

where  $\Delta y_t$  is economic growth,  $\pi_t$  is inflation,  $Policy_t^+ \max(0, \varepsilon_t)$  represents contractionary monetary policy and  $Policy_t^- = \min(0, \varepsilon_t)$  is expansionary policy. There are two reasons for including 4 lags in equations. First, for smoothing the volatility of policy stance between periods. Second, 4 lags enable to estimate impacts of both short and long-term policies.

Results of the estimation are presented in Table 1.1 in Appendix 1, which reports the presence of asymmetric effects of monetary policy on economic growth and inflation in Armenia. The cumulative impact of tight monetary policy on output growth (-0.907) is stronger compared to the easy policy (0.184). On the other hand, the response of inflation to tight monetary policy is small (-0.619) compared to expansionary one (0.843). The net impact of central bank on inflation is positive and estimated to 0.224, but the net effect on economic growth is -0.723. The results are significant at 0.05 and 0.01, respectively. Table 1.1 also presents Wald test statistics of the hypothesis that sum of coefficients on positive and negative policies equal zero. Zero hypothesis is rejected at significance level of 0.01 for 3 tests. We cannot reject only the hypothesis, that sum of coefficients on expansionary monetary policy in output growth equation is zero. Last row of the table shows t-statistics of the hypothesis that sum of coefficients on positive and negative monetary policy is zero. The hypothesis is rejected both for inflation and output growth.

Above results show, that expansionary monetary policy is less efficient to stimulate output. On the other hand, the effect of loose monetary policy on inflation is stronger. To test the robustness of results, economic growth and inflation equations are estimated, using two other measures of monetary policy. First indicator is the quarterly change in interest rate<sup>1</sup>. Second one is the deviation of interest rate from its equilibrium level, calculated by HP filter. Table 1.2 in Appendix 1 shows estimation results. Second column shows that sum of coefficients

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<sup>1</sup> See Bernanke (1990) and Laurent (1988)

on positive interest rate change is -1.297. This value is very close to the value from previous estimation based on positive residuals from interest rate estimation. The net impact of monetary policy on output growth is negative and estimated to -0.843.

Using interest rate gap as a measure of monetary policy stance, third column reports, that positive movements of interest rate are stronger to decrease the output growth (-1.267) compared to effects of negative changes in interest rate (0.438). On the other hand, the sum of the net effect of interest rate change on inflation is 0.567. The impact of the expansionary monetary policy (negative gap of interest rate) on inflation is higher than the impact of contractionary one. Statistically significant net effect is 0.357.

### 2.3. *Nonlinear VAR Model*

VAR models are standard tools in macroeconomics to analyze the impact of policy on the economy. But the standard VARs have a linear form and they are not able to generate the asymmetric response of the economy to the same size of positive and negative shock. This section constructs and estimates the nonlinear VAR model, employing the methodology developed in Kilian and Vigfusson (2011). Authors use the nonlinear VAR model to show the asymmetric response of U.S. economy to positive and negative oil price shocks.

The system is represented by following 3 equations:

$$R_t = f_0 + \sum_{i=1}^3 f_{1,i} R_{t-i} + \sum_{i=1}^3 f_{2,i} \Delta y_{t-i} + \sum_{i=1}^3 b_{3,i} \pi_{t-i} + \varepsilon_t^R \quad (2.4)$$

$$\Delta y_t = g_0 + \sum_{i=0}^3 g_{1,i} R_{t-i} + \sum_{i=0}^3 g_{2,i} R_{t-i}^+ + \sum_{i=1}^3 g_{3,i} \Delta y_{t-i} + \sum_{i=1}^3 g_{4,i} \pi_{t-i} + \varepsilon_t^y \quad (2.5)$$

$$\pi_t = h_0 + \sum_{i=0}^3 h_{1,i} R_{t-i} + \sum_{i=0}^3 h_{2,i} R_{t-i}^- + \sum_{i=1}^3 h_{3,i} \Delta y_{t-i} + \sum_{i=1}^3 h_{4,i} \pi_{t-i} + \varepsilon_t^\pi \quad (2.6)$$

The first equation is a standard linear model. Censored variables  $R_t^+$  and  $R_t^-$  are included in the output growth and inflation equations, which add nonlinearities to the system. They are given by:

$$R_t^+ = \begin{cases} R_t, & \text{if } R_t > 0 \\ 0, & \text{if } R_t \leq 0 \end{cases} \quad (2.7)$$

$$R_t^- = \begin{cases} R_t, & \text{if } R_t < 0 \\ 0, & \text{if } R_t \geq 0 \end{cases} \quad (2.8)$$

The censored variable in economic growth equation  $R_t^+$  equals to the interest rate, when the last one is positive. Otherwise, it becomes zero. Opposite to  $R_t^+$ , censored variable in inflation equation turns to zero, if the change of interest rate is not negative.

Before estimating the whole system and getting the response of the economy to monetary policy shock, the presence of asymmetry is tested. This paper employs the slope-based asymmetry test proposed in Kilian and Vigfusson (2009). The advantage of this test is that it does not require to specify the complete system. 2.5 and 2.6 equations are estimated separately. Then, the following hypotheses are tested:

$$H_0: g_{2,0} = \dots = g_{2,3} = 0 \quad (2.9)$$

$$H_0: h_{2,0} = \dots = h_{2,3} = 0 \quad (2.10)$$

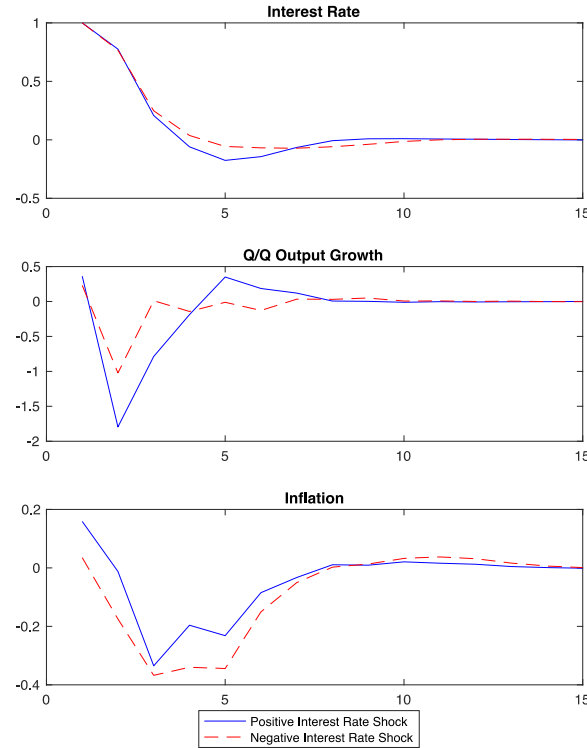
This can be calculated by Wald test with  $\chi^2(2)$  distribution. Table 3 reports results of the test. The null hypothesis is rejected with significance level of 0.05 for both equations. It means, that our system has nonlinearities.

Table 3. Results of the slope-based tests

	F-statistic	p-Value
Economic Growth (Equation 2.5)	3.0046	0.0308
Inflation (Equation 2.6)	2.6419	0.0495

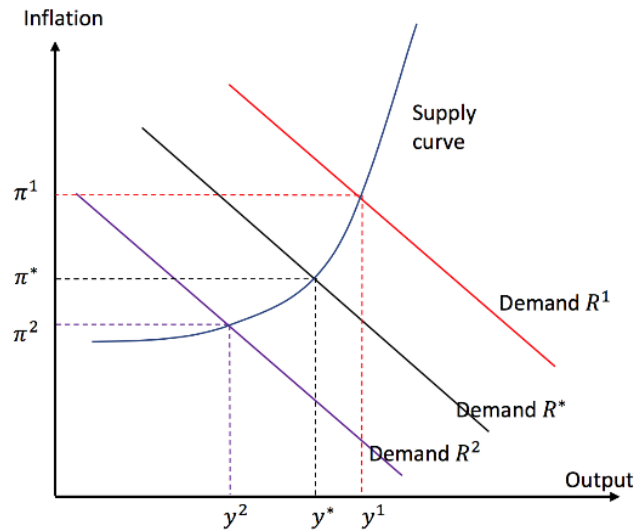
The monetary policy shock is identified following Christiano, Eichenbaum and Evans (1999). Model is estimated using Armenia's data. Figure 1 shows the response of the economy to 1% increase and decrease in interest rate. For the convenience, responses of variables to negative shock are multiplied by (-1). Simulation of the estimated model for Armenia shows, that the response of output to contractionary monetary policy is stronger compared to expansionary policy. On the other hand, low interest rate is more powerful to create inflation and high interest rate is less efficient to decrease inflation. In addition, the degree of asymmetry in economic growth is higher than the asymmetry in inflation.

Figure 1. Impulse response to interest rate shock. The responses to negative shock are shown as mirror images to facilitate the comparison



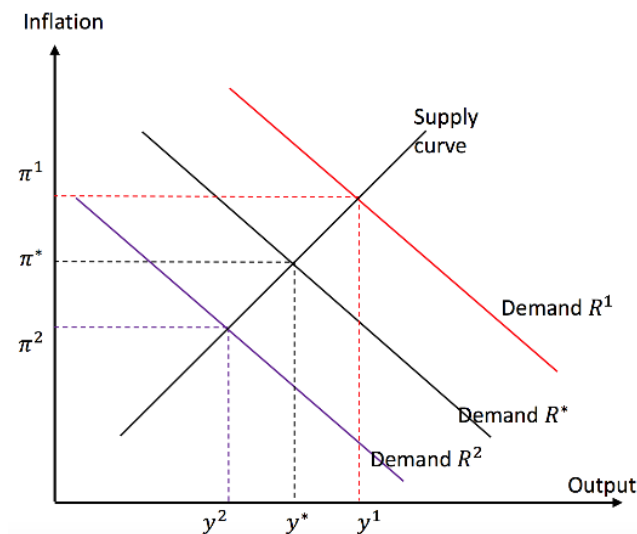
This finding is consistent with convex Phillips curve theory. Graphical illustration is presented in Figure 2. The equilibrium point of the economy is given by the interaction of demand and supply curves with the equilibrium level of output ( $y^*$ ) and inflation ( $\pi^*$ ). In this experiment, we abstract from nonlinearities of demand curve. Interest rate, controlled by central bank, shifts the demand curve. The decrease of interest rate from  $R^*$  to  $R^1$  stimulates demand. Firms, facing the growing demand for their production, increase their hiring of labor and capital to provide additional goods to the economy. In short term, capacities of the economy are restricted. So, they put much pressure on prices, but increase production relatively less. In the opposite case, when central bank increases interest rate from  $R^*$  to  $R^2$  (the size of increase and decrease in interest rate is the same), firms decrease the production more than prices, because of downward price rigidities. Taking into account the theory of convex Phillips curve, the simple graphical simulation illustrates that contractionary monetary policy reduces output more than expansionary policy stimulates it. Opposite to the response of output, the rise of inflation is stronger in a response to ease monetary policy compared to tight monetary conditions.

Figure 2. Nonlinear Phillips and Aggregate Demand Curves



Standard VARs, linearized DSGE models, which are commonly used in macroeconomic policy making institutions, predict the symmetric response of the economy to the same size of positive and negative shocks. This could be graphically illustrated by simple framework of aggregate demand and supply curves (Figure 3). The only difference from the above example is the linear Phillips curve. The increase of interest rate from  $R^*$  to  $R^2$  shifts the aggregate demand to the left and new equilibrium is formulated with  $y^2$  level of output and  $\pi^2$  inflation. The same size of expansionary monetary policy increases output to  $y^1$  and inflation to  $\pi^1$ . In the linear economy, the increase of output is equal to the decrease. The change of inflation is the same as well.

Figure 3. Linear Phillips and Aggregate Demand Curves



### **3. Small Open Economy DSGE Model for Armenia**

The empirical models, discussed in previous section, are a quite powerful tools to estimate the asymmetric response of the economy to monetary policy. However, empirical models are unable to explain the sources of asymmetries. They are not useful for policy simulations in different phases of business cycle and for the estimation of the efficiency of modest and aggressive policies. Econometric models do not explain the contribution of the specific economic theory in the creation of asymmetries. Most theoretical models are constructed in partial equilibrium framework. For example, Ball and Mankiw (1994) develop the model of costly price changing, which leads to a convex supply curve. In general equilibrium, models are usually log-linearized. So, the nonlinearities are lost, which result in a loss of the model's power to generate asymmetries and analyze the asymmetric behavior of the economy.

The literature related to the asymmetric effects of policy on the economy in general equilibrium is scarce. Wen and Wu (2008) show that first order approximation of DSGE models results in an inaccurate capture of business cycle properties. Abbritti and Fahr (2011) introduce the mechanism of downward wage rigidities in DSGE model with search and matching frictions, which enables to reproduce business cycle asymmetries observed in OECD countries. Ravn (2014) includes asymmetric monetary policy towards asset prices in DSGE framework and creates nonlinearities in the economy. Castillo and Montoro (2008) consider intertemporal nonhomotheticity in preferences of agents and induce the asymmetry in the response of the economy to positive and negative monetary policy shocks.

To explain asymmetries, affecting the transmission mechanism of monetary policy in Armenia, and to estimate the response of the economy to modest and aggressive monetary policies in different points of business cycle, this section develops micro founded theoretical model. Equations are derived from optimization problems of economic agents. To preserve sources of asymmetries, the second order Taylor approximation to the policy function is applied. Additionally, this paper develops the methodology of mixed equations (see Part 7), which enables to estimate what frictions of the theoretical model contribute to the asymmetry in transmission mechanism of policy. Without employing this methodology of mixed equations, the second order

Taylor approximation of the nonlinear model shows asymmetries of the complete economy, not allowing to get insight into the specific nonlinearities of the model.

The representative Armenian household consumes domestically produced and imported consumption goods. Each household is specialized in the particular type of labor, has some power to negotiate over wages, and supplies it to the labor organization. The labor packager combines heterogeneous labor into homogeneous one and supplies to domestic firms, which operate in monopolistic competitive environment. The discussion of labor packager is the modelling technique to introduce wage stickiness and have nonlinearities from labor market in the model. Households hold the capital stock of the economy, do investments to accumulate capital by purchasing domestically produced and imported investment goods. Households, choosing the utilization rate of capital, supply it to domestic firms, which produce intermediate goods. They use Cobb-Douglas production function by combining technology, labor and capital services. The representative firm combines varieties of intermediate inputs into the homogenous good. Then, the homogeneous good is used for consumption, investment, government expenditures and export sector. Three types of importers are operating in the economy. They import consumption goods, investment goods and goods, used in export sector. Importers import differentiated goods and set prices of goods, following price setting mechanism proposed in Calvo (1983). Central bank is operating via Taylor rule. Domestic economy is very small and does not affect the foreign economy. So, the external sector's variables are given for Armenia and they are modelled as first order autoregressive processes.

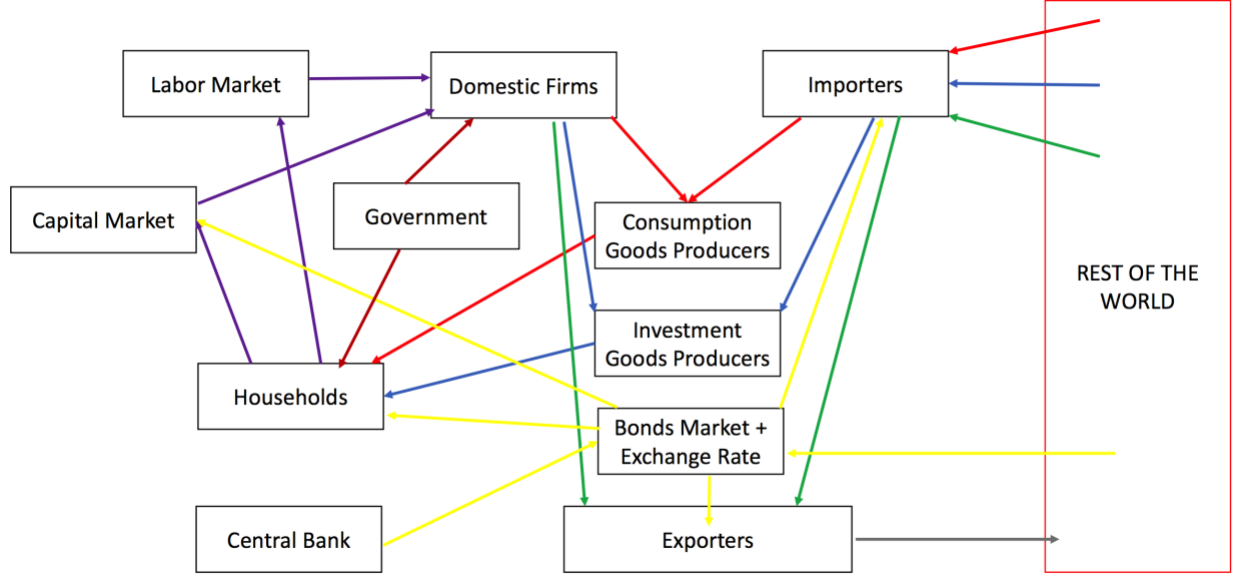
Different from this class of models developed in the literature<sup>2</sup>, this paper does not discuss the growing path of the technology, and variables are not scaled by unit root technology. There are two explanations for that. Firstly, the growing path of the model is not the relevant factor, affecting the research question of this paper. Secondly, there is a problem of linking data to the model for developing countries because of historically significant differences between

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<sup>2</sup> See Chang, Doh and Schorfheide (2006), Christiano, Trabandt and Valentin (2011), Schmitt-Grohe and Uribe (2010)

growth rates of macroeconomic variables. The schematic representation of the model is presented in Figure 4.

Figure 4. Schematic representation of the model



### 3.1. Final Consumption and Investment Goods

The consumption basket of the representative Armenian household is given by the following equation:

$$C_t = \left[ (1 - \gamma_c)^{\frac{1}{\eta_c}} C_{H,t}^{\frac{\eta_c - 1}{\eta_c}} + \gamma_c^{\frac{1}{\eta_c}} C_{F,t}^{\frac{\eta_c - 1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c - 1}} \quad (3.1)$$

where  $C_{H,t}$  and  $C_{F,t}$  are home produced and imported consumption goods,  $\gamma_c$  is the share of imported goods in household consumption bundle,  $\eta_c$  is the elasticity of substitution between two groups of goods. The aggregate consumption index of domestic ( $C_{H,t}$ ) and imported ( $C_{F,t}$ ) goods are represented by Dixit-Stiglitz constant elasticity of substitution functions:

$$C_{H,t} = \left[ \int_0^1 C(j)_{H,t}^{\frac{\varepsilon_d - 1}{\varepsilon_d}} dj \right]^{\frac{\varepsilon_d}{\varepsilon_d - 1}} \quad (3.2)$$

$$C_{F,t} = \left[ \int_0^1 C(j)_{F,t}^{\frac{\varepsilon_{c,Imp} - 1}{\varepsilon_{c,Imp}}} dj \right]^{\frac{\varepsilon_{c,Imp}}{\varepsilon_{c,Imp} - 1}} \quad (3.3)$$



where  $j \in [0, 1]$ , denotes the continuum of goods,  $\varepsilon_d$  and  $\varepsilon_{c,Imp}$  are elasticities of substitution between varieties of goods. Household minimizes expenditures per  $j$ -th domestically produced  $\int_0^1 P(j)_t C(j)_{H,t} dj$  and imported  $\int_0^1 P(j)_{F,t}^C C(j)_{F,t} dj$  consumption goods, where  $P(j)_t$  and  $P(j)_{F,t}^C$  are prices of domestic and imported  $j$ -the good, respectively. Solution of the problem of expenditures' optimal allocation gives the demand functions of the following forms:

$$C(j)_{H,t} = \left( \frac{P(j)_t}{P_t} \right)^{-\varepsilon_d} C_{H,t} \quad (3.4)$$

$$C(j)_{F,t} = \left( \frac{P(j)_{F,t}^C}{P_{F,t}^C} \right)^{-\varepsilon_{c,Imp}} C_{F,t} \quad (3.5)$$

Substituting  $C(j)_{H,t}$  and  $C(j)_{F,t}$  in equations (3.2) and (3.3) by their functional forms from (3.4) and (3.5), we can derive the composite price indexes represented by:

$$P_t = \left[ \int_0^1 P(j)_t^{1-\varepsilon_d} dj \right]^{\frac{1}{1-\varepsilon_d}} \quad (3.6)$$

$$P_{F,t}^C = \left[ \int_0^1 P(j)_{F,t}^{C 1-\varepsilon_{c,Imp}} dj \right]^{\frac{1}{1-\varepsilon_{c,Imp}}} \quad (3.7)$$

Finally, households try to minimize expenditures of two types of consumption goods. The cost minimization problem is represented by:

$$\min_{\{C_{H,t}, C_{F,t}\}} \{P_t^C C_t - P_t C_{H,t} - P_{F,t}^C C_{F,t}\} \quad (3.8)$$

where  $P_t^C$  is the consumer price index in Armenia. Solution of expenditures minimization problem gives demand functions for these two groups of goods:

$$C_{H,t} = (1 - \gamma_c) \left( \frac{P_t}{P_t^C} \right)^{-\eta_c} C_t \quad (3.9)$$

$$C_{F,t} = \gamma_c \left( \frac{P_{F,t}^C}{P_t^C} \right)^{-\eta_c} C_t \quad (3.10)$$

After permutation of (3.9) and (3.10) in (3.8), consumer price index gets the form:

$$P_t^C = \left[ (1 - \gamma_c) P_t^{(1-\eta_c)} + \gamma_c P_{F,t}^{C (1-\eta_c)} \right]^{\frac{1}{1-\eta_c}} \quad (3.11)$$

The aggregate investment good ( $I_t$ ) is the function of domestically produced ( $I_{H,t}$ ), imported ( $I_{F,t}$ ) and investment goods used in capital services ( $a(u_t)K_t$ ), which is given by the following equation:

$$I_t = \left[ (1 - \gamma_{Inv}) \frac{1}{\eta_{Inv}} I_{H,t}^{\frac{\eta_{Inv}-1}{\eta_{Inv}}} + \gamma_{Inv} \frac{1}{\eta_{Inv}} I_{F,t}^{\frac{\eta_{Inv}-1}{\eta_{Inv}}} \right]^{\frac{\eta_{Inv}}{\eta_{Inv}-1}} - a(u_t)K_t \quad (3.12)$$

where  $\gamma_{Inv}$  is the share of imported investment goods,  $\eta_{Inv}$  is the elasticity of substitution between domestic and imported investment goods,  $K_t$  is the stock of capital and  $a(u_t)$ . Following Christiano et al. (2011), this paper uses the functional form of capital utilization given by:

$$a(u_t) = \frac{1}{2} \varrho_a \varrho_b u_t^2 + \varrho_b (1 - \varrho_a) u_t + \varrho_b \left( \frac{\varrho_a}{2} - 1 \right) \quad (3.13)$$

where  $u_t$  is the utilization rate of capital,  $\varrho_a$  and  $\varrho_b$  are parameters of the function. This function is very convenient for the analysis, because it becomes zero in steady state.

Households try to minimize expenditures on investment goods. The problem is similar to cost minimization problem of consumption goods. The solution of the optimization problem yields the following demand functions:

$$I_{H,t} = (1 - \gamma_{Inv}) \left( \frac{P_t}{P_t^I} \right)^{-\eta_{Inv}} (I_t + a(u_t)K_t) \quad (3.14)$$

$$I_{F,t} = \gamma_{Inv} \left( \frac{P_{F,t}^I}{P_t^I} \right)^{-\eta_{Inv}} (I_t + a(u_t)K_t) \quad (3.15)$$

where  $P_t^I$  and  $P_{F,t}^I$  are prices of aggregate and imported investment goods, respectively. The price of  $I_t$  is the function of its components' prices, which is represented by the following:

$$P_t^I = \left[ (1 - \gamma_{Inv}) P_t^{(1-\eta_{Inv})} + \gamma_{Inv} P_{F,t}^{I(1-\eta_{Inv})} \right]^{\frac{1}{1-\eta_{Inv}}} \quad (3.16)$$

### 3.2. Households

The typical household in the economy maximizes its utility function given by:

$$U_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j \left( \omega_{t+j}^C \log(C_{t+j} - hab C_{t+j-1}) - \omega_{t+j}^N \frac{n(j)_{t+j}^{(1+\varphi)}}{1+\varphi} \right) \quad (3.17)$$

where  $E_t$  is the expectation operator conditional on information available at time  $t$ ,  $\omega_t^C$  and  $\omega_t^N$  are consumption preference and labor supply shocks.  $\beta$  denotes discount factor,  $hab$  is the

parameter of the habit in consumption,  $\varphi$  is the inverse of the Frisch elasticity of labor supply and  $n(i)_t$  represents the  $i$ -th type of labor, supplied by the household.

The maximization problem is subject to the following budget constraint written in terms of domestic goods:

$$\begin{aligned} & \frac{P_{t+j}^C}{P_{t+j}} C_{t+j} + \frac{P_{t+j}^I}{P_{t+j}} I_{t+j} + \frac{B_{t+j}}{P_{t+j}} + \frac{Ex_{t+j} F_{t+j}}{P_{t+j}} + a(u_{t+j}) K_{t+j} + T_{t+j} = \\ & = \frac{W(i)_{t+j}}{P_{t+j}} N(i)_{t+j} + u_{t+j} K_{t+j-1} R_{t+j-1}^k + \frac{R_{t+j-1} B_{t+j-1}}{P_{t+j}} + \frac{R_{t+j-1}^* Ex_{t+j} F_{t+j-1}}{P_{t+j}} + \frac{\Pi_{t+j}}{P_{t+j}} \end{aligned} \quad (3.18)$$

Left hand side of the budget constraint represents household expenditures. Each period they purchase consumption goods, investment goods, domestic ( $B_t$ ) and foreign ( $F_t$ ) bonds. They also face capital utilization costs and pay taxes ( $T_t$ ).  $Ex_t$  denotes the effective nominal exchange rate of Armenian dram. Right hand side of the constraint shows incomes. Household gets  $W(i)_t$  nominal wage for  $i$ -th type of labor.  $R_t^k$  is the rate on return of the installed capital. Households get interest payments for previously purchased domestic ( $R_{t-1} B_{t-1}$ ) and foreign bonds ( $R_{t-1}^* Ex_t F_{t-1}$ ).  $\Pi_t$  denotes dividends of firms, operating in the economy.

The second constraint in the household problem is capital accumulation law given by:

$$K_{t+j+1} = (1 - \delta) K_{t+j} + \Psi_{t+j} \left[ 1 - \Phi \left( \frac{I_{t+j}}{I_{t+j-1}} \right) \right] I_{t+j} \quad (3.19)$$

where  $\delta$  is the depreciation rate of capital,  $\Psi_{t+j}$  is the shock to marginal efficiency of investments.  $\Phi \left( \frac{I_t}{I_{t-1}} \right)$  denotes investment adjustment cost function, which has the following functional form:

$$\Phi \left( \frac{I_t}{I_{t-1}} \right) = 0.5 \left\{ e^{\sqrt{s''} \left( \frac{I_t}{I_{t-1}} - 1 \right)} + e^{-\sqrt{s''} \left( \frac{I_t}{I_{t-1}} - 1 \right)} - 2 \right\} \quad (3.20)$$

The logic of investment adjustment cost is that economy faces costs for investment changes. This form of investment adjustment costs is widely used in DSGE literature (see Christiano et al. (2005), Fernandez-Villaverde and Rubio-Ramirez (2009), Justiniano et al. (2010)). There are no costs in steady state. In addition, it becomes visible only in second order approximated model. It turns to zero when the first order approximation is applied (see Appendix 2 for more details). This investment adjustment costs function is important friction for the asymmetric reaction of capital stock to the symmetric monetary policy shock.

The household maximizes its utility subject to budget constraint and capital accumulation law, and with respect to consumption, capital stock, capital utilization rate, investment, domestic and foreign assets holdings. First order conditions of household problem are stored in Appendix 2, and they are represented by equations 1, 2, 3, 8, 11 and 12, respectively.

### 3.3. Production

The domestic homogenous good ( $Y_t$ ) is produced by competitive firm, which aggregates all the intermediate inputs by CES technology:

$$Y_t = \left[ \int_0^1 Y(j)_t^{\frac{\varepsilon_d-1}{\varepsilon_d}} dj \right]^{\frac{\varepsilon_d}{\varepsilon_d-1}} \quad (3.21)$$

The representative firm takes prices of intermediate and aggregate goods as given. Solution of the profit maximization problem gives the demand function for the  $j$ -th domestically produced intermediate good of the form:

$$Y(j)_t = \left( \frac{P(j)_t}{P_t} \right)^{-\varepsilon_d} Y_t \quad (3.22)$$

There is a continuum of intermediate goods producers represented by the unit interval  $j \in [0, 1]$ , which produce intermediate goods using Cobb-Douglas production function of the form:

$$Y(j)_t = Z_t K^{serv}(j)_t^\alpha N(j)_t^{1-\alpha} - \Phi \quad (3.23)$$

where  $K^{serv}(j)_t$  represents capital services, rented by the  $j$ -th firm,  $N(j)_t$  is homogenous labor hired by the firm,  $\alpha$  represents the share of capital and  $Z_t$  is the stationary productivity process common for all firms.  $\Phi$  denotes fixed cost of production (See Christiano et al. 2005). Firms choose capital and labor inputs, taking their prices as given. The cost minimization problem is:

$$\min_{\{K(j)_t^{serv}, N(j)_t\}} R_t^k K(j)_t^{serv} + W_t N(j)_t \quad (3.24)$$

The problem is solved subject to the production function for producing enough to meet demand. From the solution of optimization problem, real marginal cost ( $mc_t$ ) equation is represented by:

$$mc_t^d = \frac{\tau_t^d}{Z_t} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{1}{1-\alpha} \right)^{1-\alpha} (r_t^k)^\alpha (w_t)^{1-\alpha} \quad (3.25)$$

where  $\tau_t^d$  is a price mark-up shock. Small letters denote real terms of their nominal variables. The optimal allocation of resources implies capital-labor ratio to be the following:

$$\frac{K_t}{N_t} = \left[ \frac{\tau_t^d w_t}{(1 - \alpha) Z_t m c_t^d} \right]^{\frac{1}{\alpha}} \quad (3.26)$$

j-th firm has the monopolistic power to produce the differentiated good. So, it sets the price. Following Calvo price setting frictions, firm gets the signal to change prices with probability  $(1 - \theta^d)$  every period. Otherwise, it keeps the price of the good unchanged with probability  $\theta^d$ . The optimal price chosen in current period is denoted by  $P(j)_t^*$ . The j-th firm maximizes its expected profit:

$$\max_{\{P(j)_t^*\}} \sum_{j=0}^{\infty} (\beta \theta)^j E_t \left[ \Lambda_{t,t+j} \left( \frac{P(j)_t^*}{P_{t+j}} - m c_{t+j}^d \right) Y(j)_{t+j} \right] \quad (3.27)$$

subject to the demand function given by 3.22.  $\Lambda_{t,t+j}$  denotes the stochastic discount factor. Solution of the optimization problem derives the Calvo nonlinear Phillips curve of 4 equations:

$$\frac{X_{1,t}^d}{X_{2,t}^d} = \left[ \frac{1 - \theta^d (\pi_t)^{1-\varepsilon_d}}{1 - \theta^d} \right]^{\frac{1}{1-\varepsilon_d}} \quad (3.28)$$

$$X_{1,t}^d = \frac{\varepsilon_d}{\varepsilon_d - 1} \lambda_t Y_t m c_t^d + \beta \theta^d (\pi_{t+1})^{\varepsilon_d} X_{1,t+1}^d \quad (3.29)$$

$$X_{2,t}^d = \lambda_t Y_t + \beta \theta^d (\pi_{t+1})^{\varepsilon_d - 1} X_{2,t+1}^d \quad (3.30)$$

$$\tilde{p}_t = \left[ (1 - \theta^d) \left( \frac{1 - \theta^d (\pi_t)^{\varepsilon_d - 1}}{1 - \theta^d} \right)^{\frac{\varepsilon_d}{\varepsilon_d - 1}} + \theta^d \left( \frac{\tilde{p}_{t-1}}{\pi_t} \right)^{-\varepsilon_d} \right]^{-\frac{1}{\varepsilon_d}} \quad (3.31)$$

$X_{1,t}^d$  and  $X_{2,t}^d$  are auxiliary variables to make the equation of optimal price ratio convenient. Equation (3.31) shows the law of motion of price dispersion. This term is a result of reallocation of resources between firms. Price dispersion disappears in the first order approximation. It becomes relevant for the model dynamics when the second order approximation is applied.

### 3.4. Importers

Three types of importers are operating in the economy: importers of consumption goods, importers of investment goods, importers of goods used in the production of the final export goods. They have monopolistic power, repackage the homogeneous good from foreign sector

into a specialized good and supply to the domestic retailer. Domestic retailers combine continuum of specialized goods into homogenous imported good using Dixit-Stiglitz function:

$$Imp_t^i = \left[ \int_0^1 Imp_t^i(j)^{\frac{\varepsilon_{i,Imp}-1}{\varepsilon_{i,Imp}}} dj \right]^{\frac{\varepsilon_{i,Imp}}{\varepsilon_{i,Imp}-1}} \quad (3.32)$$

where  $i \in (C, I, Exp)$  is for consumption, investment and export sector goods.  $Imp_t^i$  is the aggregated import,  $\varepsilon_i$  is the elasticity of substitution between varieties of goods. Domestic aggregator is competitive and takes the price  $(P_t^i)$  of  $Imp_t^i$  and price  $(P_t^i(j))$  of  $Imp_t^i(j)$  as given. It decides how much of the j-th good to purchase. So, the profit maximization problem gives the demand for the j-th imported good of the following form:

$$Imp_t^i(j) = \left( \frac{P_t^{i,Imp}(j)}{P_t^{i,Imp}} \right)^{-\varepsilon_{i,Imp}} Imp_t^i \quad (3.33)$$

Real marginal cost of importers is given by the expression:

$$mc_t^{i,Imp} = \frac{\tau_t^{i,Imp} Ex_t P_t^*}{P_t^i} \quad (3.34)$$

where  $\tau_t^{i,Imp}$  is the mark-up on marginal costs and  $P_t^*$  is the foreign price. Importers solve profit maximization problem similar to the domestic producer's problem. As a result, three convex Phillips curves are derived from these problems. Because of price dispersion in the nonlinear model, total import concentrated for consumption, investment and export sectors  $i \in (C, I, Exp)$  is the following:

$$Imp_t = Ex_t P_t^* \left( Imp_t^C (\tilde{p}_t^{C,Imp})^{-\varepsilon_{C,Imp}} + Imp_t^I (\tilde{p}_t^{Inv,Imp})^{-\varepsilon_{I,Imp}} + Imp_t^{Exp} (\tilde{p}_t^{Exp,Imp})^{-\varepsilon_{Exp,Imp}} \right) \quad (3.35)$$

Firms set prices, following Calvo mechanism. Curvatures of Phillips curves impact on the dynamics of import sector in the second order. Linearization of the model yields to the disappearance of the curvature, which makes price dispersion terms non-relevant. Nonlinear Phillips curves of imported consumption good, imported investment good and imported good used in export sector are given in Appendix 2 by equations 20-23, 26-29 and 32-35, respectively.

### 3.5. Exporters

Foreign demand for domestically produced goods is represented by the following function:

$$Exp_t = \left( \frac{P_t^{Exp}}{Ex_t P_t^*} \right)^{-\eta_f} Y_t^* \quad (3.36)$$

where export is an increasing function of foreign demand  $Y_t^*$  and relative prices.  $\eta_f$  is the elasticity of export and  $P_t^{Exp}$  is the export price in domestic currency. The homogenous export good is produced by the representative competitive producer by combining the continuum of goods using Dixit-Stiglitz production function. On the other hand, the  $j$ -th export good is produced by monopolistic firm by combining domestically produced good with the imported one. The technology is represented by:

$$Exp_t(j) = \left[ (1 - \gamma_{exp})^{\frac{1}{\eta_{exp}}} Y^{Exp}(j)_t^{\frac{\eta_{exp}-1}{\eta_{exp}}} + \gamma_{exp}^{\frac{1}{\eta_{exp}}} Imp_t^{Exp}(j)^{\frac{\eta_{exp}-1}{\eta_{exp}}} \right]^{\frac{\eta_{exp}}{\eta_{exp}-1}} \quad (3.37)$$

where  $\gamma_{exp}$  is the share of imported goods used in the export sector,  $\eta_{exp}$  is the elasticity of substitution between imported and domestically produced goods. The exporter minimizes the cost associated with the production of export good.

$$\min_{\{Y^{Exp}(j)_t, Imp_t^{Exp}(j)\}} P_t Y^{Exp}(j)_t + P_t^{Exp, Imp} Imp_t^{Exp}(j) \quad (3.38)$$

The solution of the above optimization problem gives the marginal cost expression of the following form:

$$mc_t^{Exp} = \frac{\tau_t^{Exp} \left( (1 - \gamma_{exp}) (P_t)^{1-\eta_{exp}} + \gamma_{exp} (P_t^{Exp, Imp})^{1-\eta_{exp}} \right)^{\frac{1}{1-\eta_{exp}}}}{P_t^{Exp} Ex_t} \quad (3.39)$$

First order conditions of the same problem give demand of the exporter for the  $j$ -th domestically produced good:

$$Y^{Exp}(j)_t = (1 - \gamma_{exp}) \left[ \frac{\left( (1 - \gamma_{exp}) (P_t)^{1-\eta_{exp}} + \gamma_{exp} (P_t^{Exp, Imp})^{1-\eta_{exp}} \right)^{\frac{1}{1-\eta_{exp}}}}{P_t} \right]^{\eta_{exp}} Exp_t(j) \quad (3.40)$$

To get the aggregate input of domestically produced goods in the export sector, we need to integrate the  $Y^{Exp}(j)_t$ . In the aggregated level, the price dispersion of that sector becomes visible. Integrating equation 3.40, we get the following:

$$Y_t^{Exp} = \int_0^1 Y^{Exp}(j)_t = (1 - \gamma_{exp}) \left( 1 - \gamma_{exp} + \gamma_{exp} (p_t^{Exp, Imp})^{1-\eta_{exp}} \right)^{\frac{\eta_{exp}}{1-\eta_{exp}}} (\tilde{p}_t^{Exp})^{-\varepsilon_{Exp}} (p_t^{Exp})^{-\eta_f} Y_t^* \quad (3.41)$$

where lower letters are real variables of their nominal terms. The price dispersion in export sector is similar to the price dispersion in domestic sector, which is defined in 3.31. In the second order, price dispersion makes distortions and more inputs are needed to produce the same amount of the export good. Similarly, deriving demand for the j-th imported good used in the export sector and integrating in the unit interval, we derive the total input from the import sector of the following form:

$$Imp_t^{Exp} = \gamma_{exp} \left[ \frac{\left( 1 - \gamma_{exp} + \gamma_{exp} (p_t^{Exp, Imp})^{1-\eta_{exp}} \right)^{\frac{1}{1-\eta_{exp}}}}{p_t^{Exp, Imp}} \right]^{\eta_{exp}} (\tilde{p}_t^{Exp})^{-\varepsilon_{Exp}} (p_t^{Exp})^{-\eta_f} Y_t^* \quad (3.42)$$

### 3.6. Labor Market

The representative household supplies labor to the labor packager. The latter combines continuum of labor into a homogenous one, using the following CES technology:

$$N_t = \left[ \int_0^1 n(j)_t^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \quad (3.43)$$

Household has the monopolistic power to set wages. Following Calvo (1983),  $(1 - \theta_w)$  share of households is able to optimize wages.  $\theta_w$  parts of households set wages, taking into account the expected inflation. So, the non-optimizing household updates its wage according to the following:

$$W(j)_{t+s} = \pi_{t,t+s}^c W(j)_t \quad (3.44)$$

where  $\pi_{t,t+s}^c$  is cumulative inflation from period t to the period t+s. j-th household chooses the optimal wage  $W_t^*$  in the current period, discussing the flow of future (non-optimized, but updated



by 3.44) wages. So, the problem becomes dynamic, which is represented by the following discounted utility:

$$\sum_{s=0}^{\infty} (\beta \theta_w)^s E_t \left[ -\omega_{t+s}^N \frac{n(j)_{t+s}^{(1+\varphi)}}{1+\varphi} + \lambda_{t+s} W(j)_{t+s} n(i)_{t+s} \right] \quad (3.45)$$

On the other hand, the labor packager solves cost minimization problem, which gives the demand function for the j-th labor:

$$n(j)_{t+s} = \left( \frac{W_t^* \pi_{t,t+s}^c}{W_{t+s}} \right)^{-\varepsilon_w} N_{t+s} \quad (3.46)$$

We derive New Keynesian Phillips curve for wages by substituting the demand function into the 3.45 dynamic equation and take derivative of that with respect to optimal wage. This labor market friction adds curvatures into the model. The nonlinear Phillips curve of wages is represented by equations 55-60 in Appendix 2.

### 3.7. Aggregation

Doing aggregation of the production function, both price and wage dispersions must be taken into account.

$$Y_t = \int_0^1 Y(j)_t = \int_0^1 Z_t K^{serv}(j)_t^\alpha N(j)_t^{(1-\alpha)} - \Phi \quad (3.47)$$

Substituting the demand for the j-th good and demand for the j-th type of labor into the aggregator, we get the following production function:

$$Y_t = (\tilde{p}_t)^{\varepsilon_d} (Z_t (K_t^{serv})^\alpha (\tilde{w}_t^{\varepsilon_w} n_t)^{1-\alpha} - \Phi) \quad (3.48)$$

Price and wage distortions create inefficient allocation of resource between domestic firms, which decrease productivity or more resources are needed to produce one unit of good. The production is divided into consumption, investment, government expenditures and export.

$$Y_t = C_{H,t} + I_{H,t} + G_t + Y_t^{Exp} \quad (3.49)$$

### 3.8. Monetary Policy and Fiscal Policy

Central bank is doing inflation targeting and operates through Taylor rule of the form:

$$R_t - R^{ss} = \rho_R (R_{t-1} - R^{ss}) + (1 - \rho_R) \left( \mu_\pi (\pi_{t+1}^c - \pi^{ss}) + \mu_{gdp} \log \left( \frac{GDP_{t+1}}{GDP^{ss}} \right) \right) + \sigma_t^R \quad (3.50)$$

Interest rate has some persistence  $\rho_R$ , reacts to deviation of inflation expectation from target with  $\mu_\pi$  coefficient. Central bank responds to the output deviation from its steady state as well.

Government expenditures are exogenous, which are given by the following:

$$\log\left(\frac{G_t}{G^{ss}}\right) = \rho_g \log\left(\frac{G_{t-1}}{G^{ss}}\right) + \sigma_t^g \quad (3.51)$$

where  $G^{ss}$  is government expenditures in steady state. The latter is the average of government expenditures in GDP for the estimation period.

### 3.9. Foreign Economy

Rest of the world is given for Armenia. Foreign demand, interest rate and inflation follow first order autoregressive processes:

$$\log\left(\frac{Y_t^*}{Y^{*,ss}}\right) = \rho_{y^*} \log\left(\frac{Y_{t-1}^*}{Y^{*,ss}}\right) + \sigma_t^{Y^*} \quad (3.52)$$

$$R_t^* - R^{*,ss} = \rho_{R^*} (R_{t-1}^* - R^{*,ss}) + \sigma_t^{R^*} \quad (3.53)$$

$$\pi_t^* - \pi^{*,ss} = \rho_{\pi^*} (\pi_{t-1}^* - \pi^{*,ss}) + \sigma_t^{\pi^*} \quad (3.54)$$

where  $\rho_{y^*}$ ,  $\rho_{R^*}$  and  $\rho_{\pi^*} \in (0,1)$  are persistence coefficients and  $\sigma_t^{Y^*}$ ,  $\sigma_t^{R^*}$  and  $\sigma_t^{\pi^*}$  are independently and identically distributed shocks with zero mean.  $Y^{*,ss}$ ,  $R^{*,ss}$  and  $\pi^{*,ss}$  are steady states of foreign demand, foreign interest rate and foreign inflation, respectively. The full set of model equations is in Appendix 2.

## 4. Estimation

This paper estimates the model, developed in previous section, using the Bayesian estimation technique. The logic of the method is following: the first stage determines the approximation modal value of posterior distribution and the second stage applies MCMC technique to estimate the shape of posterior parameters distribution near the posterior mode. DSGE literature suggests to divide parameters into calibrated and estimated. Calibrated are those, which are calculated from data or strictly fixed at some point based on widely used values in literature. According to Canova (2007) and Fernandez-Villaverde et al. (2016), this strategy of estimation leads to more efficient estimation of non-calibrated parameters. Model has 15

structural shocks. This paper uses 12 Armenia's time series and 3 foreign economy variables to provide the stochastic singularity.

#### 4.1. Data

For the estimation of DSGE model, we use 15 macroeconomic data. All time series start from the first quarter of 2004 and end at the first quarter of 2017. Data on real GDP, private consumption, private investments, government expenditures, export and import are from the system of national accounts of National Statistical Service. Real effective exchange rate is taken from Central Bank of Armenia's web site. Real wage is calculated by dividing the nominal wage on CPI. Interest rate is interbank repo rate. Inflation is a quarterly percentage change in CPI. As a proxy of domestically produced good's price, quarterly percentage change in GDP deflator is used. Price of investment goods is investment deflator from the system of national accounts. Foreign inflation and GDP are calculated as a weighted average of Armenia's trading partners. Foreign interest rate is an average of USA, EU and Russia's interest rates. All time series are seasonally adjusted using X12 algorithm. To match data to the model and make it stationary, quarterly percentage changes are calculated. Appendix 4 shows model input data graphically.

Fifteen shocks in the estimation process are consumption preference, labor supply, marginal efficiency of investments, government spending, risk premium, stationary productivity, mark-up on domestically produced goods, mark-up on imported consumption goods, mark-up on imported investment goods, mark-up on imported goods used in export sector, mark-up on exported goods, monetary policy shock, foreign demand, foreign inflation and foreign interest rate. Monetary policy shock  $\sigma_t^R$  is independent and identically distributed error term. Government spending, foreign demand and foreign interest rate follow:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + (1 - \rho_\varepsilon) \varepsilon^{SS} + \sigma_t \quad (4.1)$$

where  $\varepsilon_t \in (G_t, Y_t^*, R_t^*)$ ,  $\rho_\varepsilon < 1$ , and  $\varepsilon^{SS} \in (G^{SS}, Y^{*,SS}, R^{*,SS})$  are steady state values of government expenditures, foreign demand and foreign interest rate, which are calculated from the steady state solution of the model. The remaining shocks follow first order autoregressive process of the form:

$$\varepsilon_t = \rho_\varepsilon \varepsilon_{t-1} + \sigma_t \quad (4.2)$$

where  $\varepsilon_t \in (\xi_t^c, \xi_t^n, \Psi_t, Z_t, \Omega_t, \tau_t^d, \tau_t^{c,Imp}, \tau_t^{Inv,Imp}, \tau_t^{Exp,Imp}, \tau_t^{Exp}, \pi_t^*)$  and  $\rho_\varepsilon < 1$ .

#### 4.2. Calibration

Similar to the common way of estimating DSGE models, this paper calibrates some parameters to make the estimation of structural parameters more efficient. The discount factor  $\beta$  is calibrated to 0.99, which yields annual real interest rate of 4%. The paper does not discuss the trend inflation. This implies zero inflation target or  $\pi^{target} = 1.00$ . Following the wide range of the literature (see, for example, Adolfson et al. (2007) and Christiano et al. (2011), etc.), elasticities of substitutions between varieties of domestically produced ( $\varepsilon_d$ ), imported consumption ( $\varepsilon_{c,Imp}$ ), imported investment ( $\varepsilon_{Inv,Imp}$ ), imported input in export sector ( $\varepsilon_{Exp,Imp}$ ) and exported goods ( $\varepsilon_{Exp}$ ) are calibrated to 6.

The share of government expenditures in GDP is the sample average from 2004 to 2016. The government expenditures' average share in GDP is 17% for the mentioned period. The share of labor in production function is calibrated to the value 0.45, which is the total nominal wage of all the employed divided by nominal GDP. The share of imported goods in Armenia's consumption basket is 35%. So,  $\gamma_c$  is calibrated to 0.35. The share of imported investment goods in total investments is calculated from balance of payment by dividing the nominal value of imported investment goods on total investments.  $\gamma_{Inv}$  is calibrated to 0.32 to match the average of that share for the period from 2004 to 2016. Share of imported inputs in export sector  $\gamma_{Exp}$  is calibrated to 0.2.

#### 4.3. Prior Distributions

The remaining parameters are estimated, using Bayesian estimation technique. Beta distribution is applied to parameters, which lie between zero and one. Gamma distribution is applied to parameters restricted to be positive. Means of structural shocks' standard deviations follow inverse gamma distribution.

Prior means and standard deviations of structural parameters are in Table 5.2 in Appendix 5. Following much of the literature, all the five Calvo price stickiness coefficients ( $\theta^d, \theta^{c,Imp},$

$\theta^{Inv,Imp}$ ,  $\theta^{Exp,Imp}$  and  $\theta^{Exp}$ ) and wage stickiness parameter  $\theta^w$  have beta distributions with 0.75 prior means and 0.075 standard deviations to match a yearly price and wage durations.

Inverse of the Frisch labor supply elasticity parameter  $\varphi$  is calibrated to 2 with 0.3 standard deviation. For this parameter, the above prior value is commonly used in literature (see, for example, Gali et al. (2011), Grabek et al. (2013), etc).

Following Beltran and Draper (2008), prior means and standard deviations of elasticities of substitution between home produced and imported consumption  $\eta_c$ , investment  $\eta_{Inv}$  goods are 3 and 0.45, respectively. The same prior distribution is applied for the elasticity of export  $\eta_f$  and elasticity of substitution between domestic and imported goods used in export sector  $\eta_x$ .

Habit persistence in utility function has a beta distribution with 0.5 prior mean and 0.2 standard deviation, which is the average value used in Chetty and Szeidl (2016).

The prior distribution for investment adjustment costs parameter follows gamma distribution with 9 mean and 3.1 standard deviation.

Prior of the parameter in capital utilization function follows gamma distribution with 0.2 mean and 0.075 standard deviation. We follow Smets and Wouters (2003) and Christiano et al. (2011) for defining this prior distribution.

The prior mean of persistence in interest rate  $\rho_R$  has a beta distribution with 0.7 mean and 0.12 standard deviation. In addition, responses of interest rate to inflation expectations  $\mu_\pi$  and GDP deviation from its steady state  $\mu_{gdp}$  follow gamma distribution with mean values 1.5 and 0.25, respectively.

There is a problem in identification of mark-up on wages  $\lambda_w$  parameter. So, we take the average of values used in Smets and Wouters (2007) and Gali et al. (2011). The mark-up over wages follows gamma distribution and has 1.3 prior mean and 0.15 standard deviation.

As suggested in Smets and Wouters (2007), all persistence coefficients of autoregressive processes have beta distributions with the same prior means of 0.8 and 0.085 standard deviations (see Table 5.3 in Appendix 5).

All the standard errors of structural shocks follow inverse gamma distribution (see Table 5.4 in Appendix 5). Prior means for labor supply shock and mark-up shocks are higher compared to other shocks.

#### **4.4. Posterior Estimates**

Posterior means and 90% confidence interval of estimated parameters are in the fourth, fifth and sixth columns, respectively (Tables 5.2-5.4). The estimation is obtained by running 3 parallel chains of Metropolis-Hastings algorithm with 600000 draws. The acceptance rates for 3 chains are 24.11, 25.23 and 24.48. Figure 5.2 in Appendix 5 shows the convergence diagnostics of the model's likelihood function. The blue line captures 80% interval range based on the pooled draws from all sequences. The red line shows the mean interval range based on the individual sequences' draws. The second (m2) and third (m3) rows show the estimation of the same statistics for the second and third central moments. Convergence is achieved when two lines are stabilized horizontally and should be close to each other. Figure 5.1 in Appendix 5 shows prior and posterior densities of estimated parameters. As one can see from figures, data are quite informative in obtaining the posterior distribution.

The posterior mean of price stickiness coefficient of home produced goods is 0.9, which means longer price duration. Price stickiness parameter of exported good is estimated to 0.73 capturing 4 quarters price duration. On the other hand, estimated posterior means of imported consumption goods, investment goods and imported goods used in export sector are 0.6, 0.58 and 0.49, respectively. The estimation shows, that prices of imported goods are adjusting more frequently compared to domestic prices. Castillo and Montoro (2008) show that second order approximated Calvo Phillips curve is becoming more convex when price rigidity parameter is decreasing. Estimation reports, that second order approximated Phillips curves of importers are more convex compared to domestic economy's Phillips curves. The posterior estimation of Calvo wage stickiness parameter is 0.84, which is a little bit higher than its prior mean. The estimated elasticity of substitution between varieties of labors is 7, reporting 1.16 mark-up on wages.

Elasticity of substitution between domestically produced and imported consumption goods is 1.26, which is lower compared to its prior mean. This estimation reports, that Armenian households decrease the consumption of imported consumption goods by 1.26%, when the relative price of imported goods increases by 1%. Elasticity of substitution between domestically produced and imported investment goods is 1.16 and the elasticity of substitution between

domestically produced and imported goods used in export sector is 1.4. On the other hand, the estimated elasticity of export (0.77) is smaller compared to other elasticity parameters.

Posterior mean of the habit in consumption is 0.63. Inverse elasticity of labor supply (1.88) is not very different from its prior mean, which is in line with values estimated in DSGE literature.

The estimated mean of investment adjustment costs parameter is 5.25. Posterior mean of the parameter in capital utilization function is estimated to be 0.16, which smaller than its prior mean. The value of this parameter identifies the concavity of utilization function. Small value of this parameter means less concavity of the utilization function, which decreases asymmetric adjustment of the utilization rate based on the function's properties.

The persistence parameter in Taylor rule has 0.75 posterior mean. The posterior mean of the reaction to inflation expectations is 1.56. Central bank reacts to the output deviation from its steady state with 0.14 coefficient.

Table 5.3 reports the posterior estimates of persistence coefficients in autoregressive processes. The more persistent shocks are foreign demand, foreign interest rate, risk premium, labor supply and consumption preference with posterior estimates at 0.84, 0.74, 0.72, 0.68 and 0.64, respectively. The persistence of mark-up shocks is small, which is estimated around 0.55. Supply side shocks, like productivity and marginal efficiency of investments, have low persistence. The posterior estimated persistence processes of government spending and foreign inflation are 0.62 and 0.41.

The posterior estimation results of structural shocks' standard errors are presented in Table 5.4. Labor supply shock and price mark-up shocks must be scaled by 100 to make them comparable with other structural shocks. Estimated volatilities of shocks are reasonable and economically interpretable.

## **5. Second Order Perturbation Method**

Nonlinear DSGE models are too complex to have the exact solution. That's why researchers and Central bankers use numerical approximation techniques. The common way is the log-linearization. Log-linearized model is useful for policy analysis in the neighborhood of steady state. This approximation is able to capture the overall dynamics in data and match first

(mean) and second order (standard deviation) moments in time series. However, first order approximation is not enough for policy questions regarding the asymmetric effects of monetary policy on inflation and output, the relative efficiency of modest and aggressive policies in different points of business cycle. Asymmetries are the result of model's curvature. First order approximation does not preserve the curvature of the model.

Second order approximation of the policy function is extensively applied in economics in Judd (1998). Higher order perturbation methods are studied in Collard and Juillard (2001), and Kim et al. (2002). Second order perturbation methods are widely described in Schmitt-Grohe and Uribe (2004) and Fernandez-Villaverde et al. (2016).

Equilibrium conditions of DSGE models are expressed by the following:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0 \quad (5.1)$$

where  $E_t$  is the expectation operator,  $y_t$  is the set of non-predetermined variables and  $x_t$  is the set of predetermined variables. The solution of dynamic models with rational expectations is given by:

$$y_t = g(x_t, \sigma) \quad (5.2)$$

$$x_{t+1} = h(x_t, \sigma) + \sigma \varepsilon_{t+1} \quad (5.3)$$

where  $\varepsilon_t$  is the shock with zero mean,  $\sigma$  is the known parameter. 5.2 equation requires, that non-predetermined variables are a function of predetermined variables. On the other hand, the expectation of the predetermined variable is a function of the predetermined variable at the current time. Then, the second order approximation of the  $g$  and  $h$  functions are calculated around the steady state  $x_t = x^{ss}$  and  $\sigma = 0$ .

Taylor series approximations of two solution functions are the followings:

$$\begin{aligned} g(x_t, \sigma) = & g(x^{ss}, 0) + g_x(x^{ss}, 0)(x_t - x^{ss}) + g_\sigma(x^{ss}, 0)\sigma + \frac{1}{2}g_{xx}(x^{ss}, 0)(x_t - x^{ss})^2 + \\ & + g_{x\sigma}(x^{ss}, 0)(x_t - x^{ss})\sigma + \frac{1}{2}g_{\sigma\sigma}(x^{ss}, 0)(\sigma)^2 \end{aligned} \quad (5.4)$$

$$\begin{aligned} h(x_t, \sigma) = & h(x^{ss}, 0) + h_x(x^{ss}, 0)(x_t - x^{ss}) + h_\sigma(x^{ss}, 0)\sigma + \frac{1}{2}h_{xx}(x^{ss}, 0)(x_t - x^{ss})^2 + \\ & + h_{x\sigma}(x^{ss}, 0)(x_t - x^{ss})\sigma + \frac{1}{2}h_{\sigma\sigma}(x^{ss}, 0)(\sigma)^2 \end{aligned} \quad (5.5)$$

The unknowns in above expressions are second order derivatives of functions in steady state. To find values of these derivatives, solution functions are plugged into 5.1:



$$F(x_t, \sigma) = f[g(h(x_t, \sigma) + \sigma \varepsilon_{t+1}, \sigma), g(x_t, \sigma), h(x_t, \sigma) + \sigma \varepsilon_{t+1}, x_t] = 0 \quad (5.6)$$

$F$  function is equal to zero for any  $x$  and  $\sigma$ . So, the second order derivatives of the function are equal to zero.

$$F_{xx}(x_t, \sigma) = F_{x\sigma}(x_t, \sigma) = F_{\sigma x}(x_t, \sigma) = F_{\sigma\sigma}(x_t, \sigma) = 0 \quad (5.7)$$

Second order perturbation method makes the approximation to be stable. However, it generates explosive sample paths, which are the result of additional fixed points. Additional fixed points in the system are the result of higher order terms than 2. Kim et al. (2008) develop pruning the second-order approximation. Pruning eliminates all higher order terms other than the order of solution. They show that the pruned approximation does not explode. This paper uses pruning to avoid the generations of explosive sample paths.

## 6. Sources of Curvatures in the Model

In this paper, we have several sources of asymmetries. First group of curvatures are Phillips curves for domestically produced and imported goods. Nonlinear Phillips curve for domestically produced intermediate good is expressed by equations 16-19 in Appendix 2. Calvo price setting frictions make the Phillips curve convex, which means that the trade-off between output and prices is changing along the supply curve. When the growth of the economy is accelerating, the more inflation is created. This trade-off is decreasing, when the output is below its equilibrium level. The first order approximation or log-linearization of Phillips curve yields the familiar linear New Keynesian Phillips curve of the following form:

$$\pi_t^d = \beta \pi_{t+1}^d + \frac{(1 - \theta^d)(1 - \beta \theta^d)}{\theta^d} \widetilde{mc}_t^d$$

When the economy is described by the linear supply curve, the trade-off between aggregate output and inflation is constant. This linearization cancels the potential source of the asymmetry in the model. On the other hand, price dispersion, visible in higher order approximations than one, increases the amount of capital and labor needed to produce the given level of output.

The second order approximated Phillips curve is convex. This creates nonlinear trade-off between inflation and output growth. Additionally, the convexity of Phillips curve with Calvo

price setting frictions increases with the decrease in price stickiness parameter. The estimation of structural parameters reports that prices of domestically produced goods are rigid. The Calvo parameter of domestically produced good is 0.9. High price stickiness decreases the convexity of the curve. This is because the higher price stickiness parameter decreases the responsiveness of firm's prices in optimal equilibrium to marginal costs.

The next source of asymmetry is nonlinear Phillips curve for imported consumption goods (equations 20-23 in Appendix 2). This nonlinear Phillips curve creates a nonlinear trade-off between imported consumption goods and their price. Similar to domestic economy's case, the linear approximation of 4 equations gives the forward looking linear Phillips curve:

$$\pi_t^{c,Imp} = \beta \pi_{t+1}^{c,Imp} + \frac{(1 - \theta^{c,Imp})(1 - \beta \theta^{c,Imp})}{\theta^{c,Imp}} \widetilde{mc}_t^{c,Imp}$$

This means that the convexity of the curve is lost, making the system respond to shocks symmetrically. Prices of imported goods are more flexible. The Calvo parameter of imported consumption good is estimated to 0.6, which makes the nonlinear Phillips curve more convex. The trade-off between prices and import is changing nonlinearly when moving away from steady state to both directions.

The next friction is the investment adjustment cost function given by the following:

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = 0.5 \left\{ e^{\sqrt{s''}\left(\frac{I_t}{I_{t-1}} - 1\right)} + e^{-\sqrt{s''}\left(\frac{I_t}{I_{t-1}} - 1\right)} - 2 \right\}$$

This form of investment adjustment costs is widely used in DSGE literature. The first order Taylor approximation of this function around the deterministic steady state is always zero (for more details, see Appendix 3). It becomes visible in higher order approximations. Actually, the function is symmetric, but generates asymmetries in business cycle by accelerating the relative decrease or increase in capital stock.

The next nonlinearity comes from the convex wage curve. Convex Phillips curve of wages introduces another curvature into the model. During recession households tend to decrease the employment rather than nominal wages. When in growing economy, the wealth effect is dominated over the substitution effect and households work relatively less putting pressure on nominal wages to increase. The prior belief about the elasticity of substitution between varieties of labor is 4. The posterior estimated value of this parameter is 7, which results in a higher

elasticity of substitution. Castillo et al. (2007) shows that the increase in elasticity of substitution between varieties of goods increases the convexity of Phillips curve. This high convexity results in the high asymmetric behavior of labor market in a response to shocks.

## **7. Mixed Equations Approach and Cases for Policy Analysis**

To show the asymmetric response of the economy to different shocks, second order approximation to the policy function of the model is commonly used in DSGE literature. Second order approximation of the full model is a good technique to get the asymmetric response of the economy to positive and negative shocks. However, the usefulness of the method is bounded by its disability to explain the contribution of sources of asymmetries in the creation of the asymmetric reaction of the economy to shocks. Otherwise, the contribution of the specific nonlinearity or specific sector to the creation of asymmetry is impossible to show. The second order approximation of the full model is enough for small models with one or two frictions.

The contribution of the specific friction to the asymmetry is important for policy analysis for understanding its role in the asymmetric transmission channel of monetary policy. This paper discusses the mixed equations approach. The logic of the method is that some blocks of the DSGE model are log linearized keeping other parts nonlinear. It allows to show the effect of the specific nonlinearity or the friction in a creation of the asymmetry in business cycle.

To analyze the relative importance of the specific nonlinearity to the asymmetric reaction of inflation and output to positive and negative monetary policy shocks, this paper discusses the following 7 cases using the posterior estimated coefficients for Armenia:

Case 1. The fully second order approximated model. This case analyzes the ability of the theoretical model to reproduce the empirical asymmetric responses of inflation and output to monetary policy shock.

Case 2. All frictions of the DSGE model are log-linearized. Then, the second order approximation of the policy function is applied to the remaining parts of the model. These remaining parts are household first order conditions, all marginal costs equations, production function, demand functions, market clearing condition. This exercise tests the power of small open economy model with linearized frictions to generate asymmetries.

Case 3 adds frictions from the capital market into the Case 2. Convex investment adjustment cost and quadratic utilization functions are the mentioned frictions. Simulations of this case show the power of nonlinearities in the capital market to create asymmetric business cycle.

Case 4 discusses the second order approximated domestic economy's Phillips curves. These nonlinearities are included in the Case 2. This case shows the asymmetries created by internal economy's price setting frictions.

Case 5. Labor market frictions are added to Case 2, keeping other frictions log-linearized. The purpose of this case is the calculation of labor market's contribution in a creation of asymmetries in transmission mechanism of policy.

Case 6 does simulation of the model with log-linearized external sector price setting frictions. These frictions are Phillips curves of three groups of imported goods. Second order approximation is applied to internal economy's blocks of DSGE model.

Case 7 reports simulations of the model with second order approximated external frictions and log-linearized internal economy.

## **8. Asymmetries in the Transmission Mechanism of Monetary Policy**

This section discusses one type of asymmetry, which is the reaction of the economy to the same size of positive and negative interest rate shock in the steady state. The model is calibrated based on the parameters' posterior means of the estimated DSGE model for Armenia. The above explained 7 cases are tested for the Armenian economy. Developed model has a lot of frictions, which create curvatures in the second order approximated model.

Figure 6.1 in Appendix 6 shows impulse response functions to 1% positive and negative monetary policy shocks of the second order approximated model. Simulations show, that our theoretical model is able to match the asymmetric responses of the main macroeconomic variables to expansionary and contractionary shocks. Responses of the economy are in line with empirical models' results reported in Section 2. Comparing the results of the same size of expansionary and contractionary policies, the reaction of inflation to expansionary policy is higher compared to contractionary one. On the other hand, 1% contractionary monetary policy

decreases GDP more compared to the same size of stimulating policy. The high asymmetry in inflation is created mostly by imported consumption good sector. The Phillips curve of imported consumption goods is more convex, because the Calvo stickiness coefficient of importers is small, which means that importers tend to increase prices more in an expansionary phase rather than decrease them in contractionary phase of the economy. Oppositely, when Central bank is doing expansionary policy, they increase the import of goods less making more pressures on prices. The same tendency is observed in domestic sector, but the relative size of asymmetry is less compared to import sector because prices of domestically produced goods are more sticky, which results in a less convex Phillips curve. Tight monetary policy is more effective on real sector of the economy. The reaction of consumption, investment and capital stock to tight policy is higher. The asymmetric reaction of investments creates asymmetries in investment adjustment costs, which accelerates the asymmetric response of capital stock to shocks. Instead, households react to that by changing the utilization rate asymmetrically and to opposite direction. When investments decrease and price dispersion creates more loss in productivity, households tend to increase the utilization of capital. As a result of that optimization, the utilized capital declines relatively less during contractionary monetary policy, do not allowing further loss in output.

The second step of this section switches off nonlinearities coming from frictions introduced in the model. This case loses convex Phillips curves, frictions from labor market, investment adjustment costs are becoming invisible and utilization rate of capital turns into a simple linear function instead of quadratic one. The remaining parts of model's equations are approximated up to second order. Impulse responses to 1% positive and negative shocks are shown in Figure 6.2. As one can see from results, frictionless second order approximated model is not able to capture asymmetric responses of output and inflation to monetary policy innovations. Standard assumptions are not enough for getting asymmetric responses similar to empirical model's results. This case reports symmetric responses of the main variables of the economy to the same size of positive and negative monetary policy shocks. Some little asymmetry is observed in import of consumption and investment goods, which is the result of some nonlinearities of import equations. These little asymmetries are not relevant for creating the asymmetry in inflation and output growth.

Case 3 adds the second order approximated capital market frictions into the Case 2. This model has investment adjustment cost and quadratic utilization function of capital. The economy faces higher investment adjustment costs in a response to contractionary monetary policy (see Figure 6.3). This friction creates the relatively high decline in capital and GDP. Higher decline in GDP yields higher decrease of consumption and export. This friction also creates the asymmetry in domestic economy's inflation, but the size of asymmetry is very small. The response of aggregate inflation is almost symmetric. As a result, the change in real exchange rate is symmetric. This exercise shows, that introducing nonlinear capital market frictions into the standard parts of DSGE model results in an asymmetric reaction of real sector of the economy to positive and negative monetary policy shocks to the right direction documented in theory. But the size of the asymmetry is not so strong. Capital market frictions create a little asymmetry in price setting system of domestically produced goods, but not the aggregate inflation. The coefficient in investment adjustment costs is estimated to 5.25 for the Armenian economy. The other value of this parameter can increase the role of capital market in the creation of the asymmetry in transmission mechanism of monetary policy.

Now, we make the capital market linear and add internal economy's convex Phillips curves. As it is mentioned in the model's description part, two goods are produced in the internal economy. The one is aggregate internal economy's good and the second one is exported good. This model switches on nonlinearities of Phillips curves adding curvatures to the frictionless model discussed in Case 2. These 2 frictions create asymmetries in transmission mechanism of monetary policy both in the reaction of real sector and prices. Positive monetary policy shock (see Figure 6.4 in Appendix 6) leads to the higher decrease in consumption, investments and GDP compared to the increase of these variables in a response to negative monetary policy shock. The mechanism of convex Phillips curve is working in this model. Domestic inflation increases more when the expansionary monetary policy hits the system. The size of asymmetry in domestic inflation creates the asymmetric reaction of CPI to the same size of positive and negative interest rate movements. The contribution of this price setting friction in the creation of the asymmetry in transmission mechanism of policy is relatively high compared to the asymmetry created by capital market.

Case 5 tests the contribution of model's labor market in a creation of asymmetries in the transmission mechanism of monetary policy. This case keeps other frictions of the model linear. The response of the economy to 1% positive and negative monetary policy shocks are captured in Figure 6.5. The mechanism of downward nominal rigidity works in a response to contractionary monetary policy. Wages are decreased relatively less compared to the increase of wages in a response to expansionary monetary policy. The observed asymmetry in employment is opposite. Households are willing to work less rather than face decline in nominal wages. As a result, marginal costs of firms decrease, creating fall in prices. In a response to lower employment, firms decrease the demand for capital and investments. But being in the contractionary phase, households keep the utilization rate of capital relatively high to prevent the further decline in output. The asymmetry in marginal costs creates the asymmetric reaction of domestic inflation. The asymmetry is visible also in aggregate prices, but the relative size of asymmetry is small because of the linear Phillips curve of imported goods. The tight monetary policy decreases prices less compared to the increase of prices in a response to the same size of loose monetary policy. This finding is in line with empirically reported results for the Armenian economy.

Figure 6.6 shows impulse responses of the model with second order approximated internal economy's frictions and log-linearized external sector's frictions to the same size of positive and negative monetary policy shocks. This exercise calculates the contribution of Armenia's internal economy's curvatures in a creation of asymmetries in transmission channel of monetary policy. The combination of internal frictions makes the economy respond to monetary policy more asymmetrically. The contractionary monetary policy decreases wages and return on capital more compared to increases of them in a response to the same size of expansionary policy. As a result, this model increases the asymmetry in domestic inflation, forcing the aggregate inflation to respond more asymmetrically. Real exchange rate appreciates more, which asymmetrically impacts on export sector. Asymmetries in real sector of the economy become more visible, when we have all frictions of the internal economy.

Finally, Case 7 shows the relative importance of frictions from external sector for small open economy. The positive monetary policy shock creates relatively less deflation in imported consumption goods compared to negative shock. The estimation section reports small

coefficients for both imported investment and consumption goods, which make Phillips curves of importers more convex. Much convexity of importers' Phillips curves allows them to have relatively high downward price rigidities. On the other hand, they increase prices of their goods relatively high, when expansionary monetary policy hits the economy. Importers decrease the import of consumption and investment goods relatively more during contractionary policy, do not putting much pressure on prices. As a result, they create asymmetry in aggregate consumption and investment. Households do not allow a further decline in GDP by symmetrically increasing the utilization rate of capital. This process is a result of linearized capital market frictions and symmetric reaction of physical capital's price to shocks. The response of labor market to these shocks is mixed. Firstly, wages react almost symmetrically because of the linear Phillips curve in the labor market. Secondly, the employment decreases more than it increases in a response of expansionary policy to service the aggregate production. The response of domestic inflation is symmetric, because frictions of the internal economy are log-linearized. The most visible contribution of second order approximated external sector is the creation of asymmetric response of aggregate inflation to monetary policy. The direction of the response is in line with empirical results for the Armenian economy.

This section shows, that the asymmetric responses of two important macroeconomic variables to monetary policy come from curvatures of the model created both by internal and external economy's frictions. Much of the asymmetry in output is explained by nonlinear internal frictions. The main share of asymmetry in transmission mechanism of monetary policy is created by external sector's curvatures.

## **9. Third Order Empirical Moments of Simulated Models**

Log linearized models, used in Central banks for policy analysis and forecasting, are useful tools to match means and standard deviations in data. But they generate symmetric time series with zero third order moments.

As reported in Section 2, economic growth in Armenia is negatively skewed. The skewness for the sample period from the first quarter of 2004 to the first quarter of 2017 is -1.26. Inflation is positively skewed with the value of 0.16. Finally, the asymmetry in interest rate is 0.42. This



section tests whether our theoretical model is able to generate asymmetries observed in data. Table 4 presents third order theoretical moments of inflation, output growth and interest rate, which are calculated from simulations of 7 models discussed in previous sections. Skewness is calculated from 200000 simulations.

Table 4. Skewness of Observed and Model Generated Data

	Data	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
Q/Q Inflation	0.16	0.347	0.00	0.024	0.122	0.265	0.285	0.204
Q/Q GDP Growth	-1.26	-0.935	0.042	-0.133	-0.335	-0.543	-0.636	-0.311
Interest Rate	0.42	0.563	0.023	0.091	0.181	0.331	0.388	0.135

When switching off frictions in the theoretical model, standard nonlinearities of DSGE model are not able to match skewness observed in data. Case 2 has 0 skewness for inflation, 0.042 skewness for economic growth and 0.023 skewness for interest rate. The direction of economic growth's asymmetry is opposite to what observed in data. Case 1 shows skewness, generated from second order approximated model. Directions of asymmetries are similar to macroeconomic data. Skewness of inflation, GDP growth and interest rate are 0.347, -0.935 and 0.563, respectively. The theoretical model with curvatures enables to capture not only directions of asymmetries, but also the degree of asymmetries observed in Armenia's data. Adding just one friction into a standard part of the model drives theoretical third order moments of simulated models in a right direction. The significant part of asymmetries in macroeconomic variables are the result of frictions in the internal economy (Case 6). In addition, the model with linearized internal economy and second order approximated external frictions (Case 7) generate 0.204 skewness in inflation, -0.311 skewness in economic growth and 0.135 skewness in interest rate, which are in line with empirical moments of Armenia's data.

## 10. The Efficiency of Monetary Policy in Growing Economies and Recessions

In second order, the efficiency of monetary policy on the economy might be different in different phases of business cycle. The other type of asymmetry arises when the economy is in expansion or in recession. It also depends whether business cycle is driven by demand or supply

shocks. In this paper, demand shocks are those shocks, which move output and inflation to the same direction. Responses of output and inflation to supply shocks are opposite.

Table 5 presents forecast error variance decomposition of inflation and GDP growth for periods 1, 5, 20 and 100 based on the posterior mean of structural parameters and standard deviations of structural shocks of the estimated model. In the short run, demand shocks explain around 52% variation of inflation. The contribution of demand shocks decreases to 46% in long run. On the other hand, supply shocks explain 47% of inflation's variation in the short run. In the long run, the variation of inflation is mostly driven by supply side shocks (around 52%). The contribution of monetary policy does not face sizable changes from short to long run. The relatively less variation of economic growth is driven by demand shocks. The size of the latter is 38% in the short run and decreases to 36% in longer horizons. Half of economic growth's variation is driven by supply shocks. The contribution of monetary policy shock decreases from 13% in the first period to 11% in longer horizons. Variance decomposition reports, that both demand and supply side shocks are important drivers of Armenia's business cycle.

Table 5. Variance Decomposition of Inflation and Economic Growth

	Period 1		Period 5		Period 20		Period 100	
	Inflation	Economic Growth	Inflation	Economic Growth	Inflation	Economic Growth	Inflation	Economic Growth
Demand	51.94	38.29	46.16	36.41	46.20	36.04	46.10	36.02
Supply	46.72	48.55	52.7	52.08	52.69	52.38	52.79	52.41
Monetary	1.34	13.16	1.14	11.51	1.11	11.58	1.11	11.57

This section estimates the efficiency of monetary policy in growing economy and recession. Growing economy is defined as a positive deviation of GDP from its steady state by 5%. Recession is a state of the economy, when GDP is 5 % below from its steady state. In addition, having the importance of supply and demand side shocks in explaining the variation of business cycle in Armenia, we consider demand and supply driven economies separately. In demand driven economy, we estimate the relative asymmetry of modest and aggressive expansionary policies in recession, and the relative efficiency of modest and aggressive policies in the growing economy. Modest policy means 1% monetary policy shock. 2% monetary policy shock is the

aggressive response. Graphs of aggressive responses are multiplied by 0.5 to make the comparison visible. When considering monetary policy during supply driven business cycles, the contractionary policy is discussed both during booms and recessions. The reason is that inflation and output respond to supply side shocks in an opposite direction.

Figure 7.1 in Appendix 7 shows responses of the economy to modest monetary policy in demand driven growing economy and recession. In demand driven growing economy we discuss positive interest rate shock, and the negative interest rate is applied to the recession. When the positive business cycle is driven by demand side shock, the efficiency of monetary policy is relatively more to impact on inflation rather than on output. In a demand driven growing economies, firms have hired additional labor and capital, putting pressure on inflation. In a demand driven growing economy, the aggregate demand intersects the aggregate supply in a steeper region. On the other hand, demand driven recession intersects the aggregate supply curve in flatter part. As a result, expansionary policy shock stimulates economy more effectively compared to the contractionary policy in preventing the stimulating economy. Instead, the positive policy is effective for controlling inflation in a demand driven growing economy.

The next exercise (see Figure 7.2 in Appendix 7) tests the relative efficiency of aggressive monetary policy in a demand driven growing economy, which is another type of asymmetry. By responding to demand driven growing economy aggressively, the marginal efficiency of preventing the inflation is decreasing. On the other hand, the impact of aggressive policy on the economy's real sector is strong. This type of asymmetry is a result of the time varying trade-off between inflation and output along the business cycle. Aggressive response increases the amount of output needed to cut for decreasing the given inflation. These effects come from curvatures of the model. The consumption goods importers have a more convex Phillips curve, which results in a less decline in prices and more decline in import compared to the domestic producer.

When the economy is in demand driven recession (see Figure 7.3), aggressive expansionary monetary policy leads to relative decline in the efficiency of stimulating the economy's real sector. In a recession, aggregate demand intersects the aggregate supply in a flatter region, which results in a high trade-off between output and inflation. Moving along the

business cycle to the direction of steady state, this trade-off decreases, and less output is needed to have the additional inflation. The policy experiment concludes, that aggressive policy accelerates the increase in inflation and decreases the efficiency of stimulating output in a demand driven recession.

Next 3 policy simulations are done in a supply driven business cycle environments. Only the efficiency of positive monetary policy shock is estimated, because inflation and output move to opposite directions during supply shocks. Figure 7.4 shows impulse response functions of 1% positive monetary policy shock in supply driven expansion and recession. Being in growing economy, tight monetary policy impacts on the real economy more strongly, because no additional resources are hired for the expansion of GDP. Opposite to expansion, supply driven recession intersects the aggregate supply curve with aggregate demand at its steeper region. As a result, firms do not decrease so much labor and capital inputs. Instead, they put relatively much pressure on inflation. The relatively high asymmetry is observed in imported sector because of the high convexity of imported consumption good's Phillips curve.

The next policy experiment shows the relative asymmetry caused by modest and aggressive monetary policies in supply driven growing economy (see Figure 7.5). The modest response moves the aggregate demand of the economy to more flat region of supply curve. As a consequence of that, the further policy tightening has the diminishing relative impact on inflation. The existence of downward wage rigidities in labor market forces wages to decline less on margin. Instead, decreases of labor and capital inputs accelerate. In summary, the relative response of economic growth to aggressive policy increases nonlinearly. On the other hand, the aggressive monetary policy does not have a significant marginal effect on inflation.

When the recession is driven by supply side shocks, directions of real sector variables' responses are the same compared to previous experiment (see Figure 7.6). Differences arise in sizes of asymmetries. Firms reduce output relatively less compared to the case of expansion. The reason is that the economy appears in a steeper point of aggregate supply, and the relative pressure on real sector is reduced compared to supply driven expansion. But the size of asymmetry in inflation increases. Inflation decreases relatively less in a response to aggressive monetary policy.

## 11. Conclusion

This paper empirically shows the presence of asymmetries in the transmission mechanism of monetary policy. Two stage estimation reports, that contractionary monetary policy has more powerful effects on output compared to the same size of expansionary policy. The opposite results are estimated for the impact on inflation. Furthermore, nonlinear VAR model is constructed. The estimated model for Armenia reports the asymmetric reaction of output and inflation to the same size of positive and negative monetary policy shocks.

The sources of asymmetries are not visible in reduced form econometric models. The paper develops New Keynesian DSGE model to explain reported asymmetries theoretically. New Keynesian frictions add curvatures into the model. The model is estimated for the Armenian economy. The estimation's diagnostic measures indicate the well quality of estimation.

A number of key results have emerged from the analysis. Firstly, convexities of importers' Phillips curves are higher compared to the convexity of internal Phillips curve. Secondly, capital market frictions contribute to the asymmetry in real sector of the economy more than to the asymmetry of inflation. Thirdly, internal economy's price setting frictions have the significant impact both on the asymmetry in domestic inflation and real sector variables, like consumption, investment and capital stock. Fourthly, inclusion of labor market rigidities into the model with nonlinear internal economy accelerates the asymmetry in domestic inflation and output to the response of positive and negative monetary policy shocks. Fifthly, the significant part of asymmetric response of inflation is the result of nonlinear import sector's curvatures. Sixthly, more than 50% of the Armenian business cycle is driven by supply side shocks. Seventhly, monetary tightening is more effective to decrease inflation in demand driven growing economy compared to the creation of inflation by the same size of expansionary monetary policy in demand driven recession. Eighthly, the tight monetary policy is more effective to control inflation in supply driven recession than in supply driven expansions. Ninthly, the efficiency of controlling inflation decreases with the increase in reaction of tight monetary policy in demand driven growing economy. Tenthly, aggressive expansionary policy raises the efficiency of stimulating inflation in demand driven recessions and diminishes the stimulation of economy's real sector. On the other hand, the aggressive monetary policy does not create the significant asymmetry in

inflation in supply driven expansions. Finally, strong monetary policy in recessions, caused by supply side shocks, diminishes the efficiency to control inflation.

The developed mixed equations methodology finds out the contribution of the particular nonlinearity of the model in a creation of asymmetry in transmission mechanism of monetary policy. For testing the performance of the model to match third order moments in Armenia's data, stochastic simulations of the estimated models are done based on the posterior estimated means of structural parameters and shocks' standard deviations. Having only one nonlinear friction, the reported skewness of the model main variables become in line with empirical results from data. The estimated theoretical model is able to match both directions and sizes of asymmetries, observed in macroeconomic variables of Armenia.

## Appendix 1. Estimation Results of Empirical Models

Table 1.1. Estimation of the output growth and inflation equations

	Q/Q Output Growth	Q/Q Inflation
<i>Q/Q Output Growth (-1)</i>	0.206*** (0.057)	
<i>Q/Q Inflation (-1)</i>		0.337*** (0.022)
<i>Policy(-1)<sup>+</sup></i>	-0.724*** (0.47)	-0.096* (0.051)
<i>Policy(-2)<sup>+</sup></i>	0.661*** (0.041)	0.051 (0.063)
<i>Policy(-3)<sup>+</sup></i>	-1.15*** (0.106)	-0.056 (0.045)
<i>Policy(-4)<sup>+</sup></i>	0.306** (0.132)	-0.517*** (0.075)
<i>Policy(-1)<sup>-</sup></i>	-0.197* (0.103)	0.323*** (0.043)
<i>Policy(-2)<sup>-</sup></i>	-1.014*** (0.186)	-0.354*** (0.102)
<i>Policy(-3)<sup>-</sup></i>	0.697*** (0.171)	0.158** (0.06)
<i>Policy(-4)<sup>-</sup></i>	0.698*** (0.097)	0.714*** (0.055)
Constant	0.056 (0.182)	0.049*** (0.005)
<i>Sum(Policy<sup>+</sup>)<sup>1</sup></i>	-0.907*** (0.186)	-0.619*** (0.075)
<i>Sum(Policy<sup>-</sup>)<sup>2</sup></i>	0.184 (0.193)	0.843*** (0.154)
<i>Sum(Policy<sup>+</sup>) + Sum(Policy<sup>-</sup>)<sup>3</sup></i>	-0.723*** (0.215)	0.224** (0.104)
<i>Policy<sup>+</sup> = 0<sup>4</sup></i>	185.577***	67.32***
<i>Policy<sup>-</sup> = 0<sup>5</sup></i>	0.902	29.869***
<i>Policy<sup>+</sup> + Policy<sup>-</sup> = 0<sup>6</sup></i>	11.224***	4.596**

Note. Standard errors are in parentheses.

\*, \*\* and \*\*\* significant at 0.1, 0.05 and 0.01 respectively

<sup>1</sup> sum of the coefficients on contractionary monetary policy

<sup>2</sup> sum of the coefficients on expansionary monetary policy

<sup>3</sup> sum of the net effect of monetary policy

<sup>4</sup> Wald test of the hypothesis that the sum of coefficients on contractionary monetary policy equals zero,  $\chi^2(2)$

<sup>5</sup> Wald test of the hypothesis that the sum of coefficients on expansionary monetary policy equals zero,  $\chi^2(2)$

<sup>6</sup> t-statistics of the hypothesis that the sum of coefficients on *Policy<sup>+</sup>* and *Policy<sup>-</sup>* equals zero

Table 1.2. Estimation of the output growth and inflation equations (other measures)

	Q/Q Output Growth (Gap Equation)	Q/Q Output Growth (Interest Rate Change)	Q/Q Inflation (Gap Equation)	Q/Q Inflation (Interest Rate Change)
<i>Q/Q Output Growth (-1)</i>	0.165*** (0.049)	0.134*** (0.053)		
<i>Q/Q Inflation (-1)</i>			0.203*** (0.067)	0.154*** (0.018)
<i>Policy(-1)<sup>+</sup></i>	0.182 (0.196)	-1.076*** (0.133)	0.279* (0.161)	0.160*** (0.026)
<i>Policy(-2)<sup>+</sup></i>	-0.526*** (0.066)	0.509** (0.232)	0.221* (0.114)	-0.265*** (0.070)
<i>Policy(-3)<sup>+</sup></i>	-0.465*** (0.097)	-0.519*** (0.141)	-0.686*** (0.113)	-0.185*** (0.056)
<i>Policy(-4)<sup>+</sup></i>	-0.489*** (0.113)	-0.180* (0.097)	-0.436*** (0.081)	-0.351*** (0.092)
<i>Policy(-1)<sup>-</sup></i>	-1.440*** (0.278)	-0.095 (0.084)	0.333*** (0.120)	-0.073 (0.054)
<i>Policy(-2)<sup>-</sup></i>	0.849*** (0.251)	-0.261 (0.162)	-1.159*** (0.247)	-0.099 (0.065)
<i>Policy(-3)<sup>-</sup></i>	2.115*** (0.491)	0.362*** (0.089)	0.804** (0.388)	0.228*** (0.043)
<i>Policy(-4)<sup>-</sup></i>	-1.070*** (0.201)	0.432*** (0.099)	1.210*** (0.177)	0.942*** (0.159)
Constant	0.107 (0.336)	0.044 (0.126)	0.013 (0.107)	0.068 (0.092)
<i>Sum(Policy<sup>+</sup>)<sup>1</sup></i>	-1.297*** (0.096)	-1.267*** (0.197)	-0.621*** (0.186)	-0.640*** (0.091)
<i>Sum(Policy<sup>-</sup>)<sup>2</sup></i>	0.454** (0.223)	0.438* (0.220)	1.188*** (0.276)	0.998*** (0.072)
<i>Sum(Policy<sup>+</sup>) + Sum(Policy<sup>-</sup>)<sup>3</sup></i>	-0.843*** (0.286)	-0.829*** (0.194)	0.567** (0.258)	0.357*** (0.118)
<i>Policy<sup>+</sup> = 0<sup>4</sup></i>	179.098***	41.311***	11.131***	49.615***
<i>Policy<sup>-</sup> = 0<sup>5</sup></i>	4.128**	3.940*	18.456***	189.828***
<i>Policy<sup>+</sup> + Policy<sup>-</sup> = 0<sup>6</sup></i>	8.660***	18.186***	2.197**	3.007***

Note. Standard errors are in parentheses.

\*, \*\* and \*\*\* significant at 0.1, 0.05 and 0.01 respectively

<sup>1</sup> sum of the coefficients on contractionary monetary policy

<sup>2</sup> sum of the coefficients on expansionary monetary policy

<sup>3</sup> sum of the net effect of monetary policy

<sup>4</sup> Wald test of the hypothesis that the sum of coefficients on contractionary monetary policy equals zero,  $\chi^2(2)$

<sup>5</sup> Wald test of the hypothesis that the sum of coefficients on expansionary monetary policy equals zero,  $\chi^2(2)$

<sup>6</sup> t-statistics of the hypothesis that the sum of coefficients on *Policy<sup>+</sup>* and *Policy<sup>-</sup>* equals zero



## Appendix 2. Model Equations

1. Household FOC on consumption

$$p_t^c \lambda_t = \frac{\xi_t^c}{C_t - hC_{t-1}} - \frac{\beta h \xi_{t+1}^c}{C_{t+1} - hC_t}$$

2. Household FOC on capital

$$\lambda_t = \beta \lambda_{t+1} \frac{R_{t+1}^k}{\pi_{t+1}^d}$$

3. Household FOC on capital utilization

$$r_t^k = p_t^{Inv} (\varrho^a \varrho^b u_t + \varrho^b (1 - \varrho^a))$$

4. Investment adjustment costs function

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = 0.5 \left\{ e^{\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} + e^{-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} - 2 \right\}$$

5. Derivative of investment adjustment costs function with respect to  $\frac{I_t}{I_{t-1}}$

$$\Phi'\left(\frac{I_t}{I_{t-1}}\right) = 0.5 \sqrt{S''} \left\{ e^{\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} - e^{-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} \right\}$$

6. Capital utilization function

$$a(u_t) = \frac{1}{2} \varrho^a \varrho^b u_t^2 + \varrho^b (1 - \varrho^a) u_t + \varrho^b \left( \frac{\varrho^a}{2} - 1 \right)$$

7. Return on investment done in period t

$$R_{t+1}^k = \frac{\pi_{t+1}^d}{p_t^k} [u_{t+1} r_{t+1}^k - p_{t+1}^{Inv} a(u_{t+1}) + (1 - \delta) p_{t+1}^k]$$

8. Household FOC on investment

$$\lambda_t p_t^{Inv} = \lambda_t p_t^k \Psi_t \left[ 1 - \Phi\left(\frac{I_t}{I_{t-1}}\right) - \Phi'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} \right] + \beta \lambda_{t+1} p_{t+1}^k \Psi_{t+1} \Phi'\left(\frac{I_{t+1}}{I_t}\right) \left[ \frac{I_{t+1}}{I_t} \right]^2$$

9. Capital accumulation equation

$$\bar{K}_{t+1} = (1 - \delta)\bar{K}_t + \Psi_t \left[ 1 - \Phi \left( \frac{I_t}{I_{t-1}} \right) \right] I_t$$

10. Utilized capital

$$K_t = u_t \bar{K}_t$$

11. Household FOC on domestic assets

$$\lambda_t = \beta \lambda_{t+1} \frac{R_t}{\pi_{t+1}^d}$$

12. Household FOC on foreign assets

$$\lambda_t = \beta \lambda_{t+1} \frac{\Delta E_{t+1} R_t^* \Omega_t}{\pi_{t+1}^d}$$

13. Production side of the GDP

$$Y_t = \tilde{p}_t^{d\varepsilon_d} (Z_t K_t^\alpha N_t^{(1-\alpha)} - X)$$

14. Marginal costs in the domestic intermediate good production sector

$$mc_t^d = \frac{\tau_t^d}{Z_t} \left( \frac{1}{\alpha} \right)^\alpha \left( \frac{1}{1-\alpha} \right)^{1-\alpha} (r_t^k)^\alpha (w_t)^{1-\alpha}$$

15. Marginal costs as a function of capital-labor ratio

$$mc_t^d = \frac{\tau_t^d w_t}{(1-\alpha) Z_t \left( \frac{K_t}{N_t} \right)^\alpha}$$

16. First auxiliary variable of intermediate good Phillips curve

$$X_{2,t}^d = \lambda_t Y_t + \beta \theta^d (\pi_{t+1})^{\varepsilon_d - 1} X_{2,t+1}^d$$

17. Second auxiliary variable of intermediate good Phillips curve

$$X_{1,t}^d = \frac{\varepsilon_d}{\varepsilon_d - 1} \lambda_t Y_t mc_t^d + \beta \theta^d (\pi_{t+1})^{\varepsilon_d} X_{1,t+1}^d$$

18. Optimal ratio in intermediate good production sector

$$\frac{X_{1,t}^d}{X_{2,t}^d} = \left[ \frac{1 - \theta^d (\pi_t)^{1-\varepsilon_d}}{1 - \theta^d} \right]^{\frac{1}{1-\varepsilon_d}}$$

19. Price distortion in intermediate good production sector

$$\tilde{p}_t = \left[ (1 - \theta^d) \left( \frac{1 - \theta^d (\pi_t)^{\varepsilon_d-1}}{1 - \theta^d} \right)^{\frac{\varepsilon_d}{\varepsilon_d-1}} + \theta^d \left( \frac{\tilde{p}_{t-1}}{\pi_t} \right)^{-\varepsilon_d} \right]^{-\frac{1}{\varepsilon_d}}$$

20. First auxiliary variable of imported consumption goods Phillips curve

$$X_{2,t}^{c,Imp} = \lambda_t C_t^{Imp} p_t^{c,Imp} + \beta \theta^{c,Imp} (\pi_{t+1}^{c,Imp})^{\varepsilon_{c,Imp}-1} X_{2,t+1}^{c,Imp}$$

21. Second auxiliary variable of imported consumption goods Phillips curve

$$X_{1,t}^{c,Imp} = \frac{\varepsilon_{c,Imp}}{\varepsilon_{c,Imp} - 1} \lambda_t C_t^{Imp} p_t^{c,Imp} m c_t^{c,Imp} + \beta \theta^{c,Imp} (\pi_{t+1}^{c,Imp})^{\varepsilon_{c,Imp}} X_{1,t+1}^{c,Imp}$$

22. Optimal ratio in imported consumption goods

$$\frac{X_{1,t}^{c,Imp}}{X_{2,t}^{c,Imp}} = \left[ \frac{1 - \theta^{c,Imp} (\pi_t^{c,Imp})^{1-\varepsilon_{c,Imp}}}{1 - \theta^{c,Imp}} \right]^{\frac{1}{1-\varepsilon_{c,Imp}}}$$

23. Price distortion of imported consumption goods

$$\tilde{p}_t^{c,Imp} = \left[ (1 - \theta^{c,Imp}) \left( \frac{1 - \theta^{c,Imp} (\pi_t^{c,Imp})^{\varepsilon_{c,Imp}-1}}{1 - \theta^{c,Imp}} \right)^{\frac{\varepsilon_{c,Imp}}{\varepsilon_{c,Imp}-1}} + \theta^{c,Imp} \left( \frac{\tilde{p}_{t-1}^{c,Imp}}{\pi_t^{c,Imp}} \right)^{-\varepsilon_{c,Imp}} \right]^{-\frac{1}{\varepsilon_{c,Imp}}}$$

24. Import of consumption goods

$$C_t^{Imp} = \gamma_c \left( \frac{p_t^c}{p_t^{c,Imp}} \right)^{\eta_c} C_t$$

25. Marginal cost of imported consumption goods

$$m c_t^{c,Imp} = \frac{\tau_t^{c,Imp} q_t p_t^c}{p_t^{c,Imp}}$$

26. First auxiliary variable of imported investment goods Phillips curve

$$X_{2,t}^{Inv,Imp} = \lambda_t I_t^{Imp} p_t^{c,Imp} + \beta \theta^{Inv,Imp} (\pi_{t+1}^{Inv,Imp})^{\varepsilon_{Inv,Imp}-1} X_{2,t+1}^{Inv,Imp}$$

27. Second auxiliary variable of imported investment goods Phillips curve

$$X_{1,t}^{Inv,Imp} = \frac{\varepsilon_{Inv,Imp}}{\varepsilon_{Inv,Imp} - 1} \lambda_t I_t^{Imp} p_t^{Inv,Imp} m c_t^{Inv,Imp} + \beta \theta^{Inv,Imp} (\pi_{t+1}^{Inv,Imp})^{\varepsilon_{Inv,Imp}} X_{1,t+1}^{Inv,Imp}$$

28. Price distortion of imported investment goods

$$\tilde{p}_t^{Inv,Imp} = \left[ (1 - \theta^{Inv,Imp}) \left( \frac{1 - \theta^{Inv,Imp} (\pi_t^{Inv,Imp})^{\varepsilon_{Inv,Imp}-1}}{1 - \theta^{Inv,Imp}} \right)^{\frac{\varepsilon_{Inv,Imp}}{\varepsilon_{Inv,Imp}-1}} + \theta^{Inv,Imp} \left( \frac{\tilde{p}_{t-1}^{Inv,Imp}}{\pi_t^{Inv,Imp}} \right)^{-\varepsilon_{Inv,Imp}} \right]^{-\frac{1}{\varepsilon_{Inv,Imp}}}$$

29. Optimal ratio in imported investment goods

$$\frac{X_{1,t}^{Inv,Imp}}{X_{2,t}^{Inv,Imp}} = \left[ \frac{1 - \theta^{Inv,Imp} (\pi_t^{Inv,Imp})^{1-\varepsilon_{Inv,Imp}}}{1 - \theta^{Inv,Imp}} \right]^{\frac{1}{1-\varepsilon_{Inv,Imp}}}$$

30. Marginal cost of imported investment goods

$$m c_t^{Inv,Imp} = \frac{\tau_t^{Inv,Imp} q_t p_t^c}{p_t^{Inv,Imp}}$$

31. Import of investment goods

$$I_t^{Imp} = \gamma_{Inv} \left( \frac{p_t^{Inv}}{p_t^{Inv,Imp}} \right)^{\eta_{Imp}} [I_t + a(u_t) \bar{K}_t]$$

32. First auxiliary variable of imported goods used in export sector Phillips curve

$$X_{2,t}^{Exp,Imp} = \lambda_t Exp_t^{Imp} p_t^{Exp,Imp} + \beta \theta^{Exp,Imp} (\pi_{t+1}^{Exp,Imp})^{\varepsilon_{Exp,Imp}-1} X_{2,t+1}^{Exp,Imp}$$

33. Second auxiliary variable of imported goods used in export sector Phillips curve

$$X_{1,t}^{Exp,Imp} = \frac{\varepsilon_{Exp,Imp}}{\varepsilon_{Exp,Imp} - 1} \lambda_t Exp_t^{Imp} p_t^{Exp,Imp} mc_t^{Exp,Imp} + \beta \theta^{Exp,Imp} (\pi_{t+1}^{Exp,Imp})^{\varepsilon_{Exp,Imp}} X_{1,t+1}^{Exp,Imp}$$

34. Optimal ratio in imported goods used in export sector

$$\frac{X_{1,t}^{Exp,Imp}}{X_{2,t}^{Exp,Imp}} = \left[ \frac{1 - \theta^{Exp,Imp} (\pi_t^{Exp,Imp})^{\varepsilon_{Exp,Imp}}}{1 - \theta^{Exp,Imp}} \right]^{\frac{1}{1 - \varepsilon_{Exp,Imp}}}$$

35. Price distortion of imported goods used in export sector

$$\tilde{p}_t^{Exp,Imp} = \left[ (1 - \theta^{Exp,Imp}) \left( \frac{1 - \theta^{Exp,Imp} (\pi_t^{Exp,Imp})^{\varepsilon_{Exp,Imp} - 1}}{1 - \theta^{Exp,Imp}} \right)^{\frac{\varepsilon_{Exp,Imp}}{\varepsilon_{Exp,Imp} - 1}} + \theta^{Exp,Imp} \left( \frac{\tilde{p}_{t-1}^{Exp,Imp}}{\pi_t^{Exp,Imp}} \right)^{-\varepsilon_{Exp,Imp}} \right]^{-\frac{1}{\varepsilon_{Exp,Imp}}}$$

36. Marginal cost of imported goods used in export sector

$$mc_t^{Exp,Imp} = \frac{\tau_t^{Exp,Imp} q_t p_t^c}{p_t^{Exp,Imp}}$$

37. Import of goods used in export sector

$$Imp_t^{Exp} = \gamma_{Exp} \left[ \frac{(\gamma_{Exp} (p_t^{Exp,Imp})^{(1 - \eta_{Exp})} + 1 - \gamma_{Exp})^{\frac{1}{1 - \eta_{Exp}}}}{p_t^{Exp,Imp}} \right]^{\eta_{Exp}} (\tilde{p}_t^{Exp})^{-\varepsilon_{Exp}} (p_t^{Exp})^{-\eta_f} Y_t^*$$

38. First auxiliary variable of export Phillips curve

$$X_{2,t}^{Exp} = \lambda_t q_t p_t^c p_t^x Exp_t + \beta \theta^{Exp} (\pi_{t+1}^{Exp})^{\varepsilon_{Exp} - 1} X_{2,t+1}^{Exp}$$

39. Second auxiliary variable of export Phillips curve

$$X_{1,t}^{Exp} = \frac{\varepsilon_{Exp}}{\varepsilon_{Exp} - 1} \lambda_t q_t p_t^c p_t^x Exp_t mc_t^{Exp} + \beta \theta^{Exp} (\pi_{t+1}^{Exp})^{\varepsilon_{Exp}} X_{1,t+1}^{Exp}$$

40. Optimal ratio in export sector

$$\frac{X_{1,t}^{Exp}}{X_{2,t}^{Exp}} = \left[ \frac{1 - \theta^{Exp} (\pi_t^{Exp})^{1-\varepsilon_{Exp}}}{1 - \theta^{Exp}} \right]^{\frac{1}{1-\varepsilon_{Exp}}}$$

41. Price distortion of imported goods used in export sector

$$\tilde{p}_t^{Exp} = \left[ (1 - \theta^{Exp}) \left( \frac{1 - \theta^{Exp} (\pi_t^{Exp})^{\varepsilon_{Exp}-1}}{1 - \theta^{Exp}} \right)^{\frac{\varepsilon_{Exp}}{\varepsilon_{Exp}-1}} + \theta^{Exp} \left( \frac{\tilde{p}_{t-1}^{Exp}}{\pi_t^{Exp}} \right)^{-\varepsilon_{Exp}} \right]^{-\frac{1}{\varepsilon_{Exp}}}$$

42. Marginal cost of exporter

$$mc_t^{Exp} = \frac{\tau_t^{Exp}}{q_t p_t^c p_t^x} \left[ \gamma_{Exp} (p_t^{Exp, Imp})^{1-\eta_x} + 1 - \gamma_{Exp} \right]^{\frac{1}{1-\eta_x}}$$

43. Foreign demand for domestic export

$$Exp_t = (p_t^x)^{-\eta_f} Y_t^*$$

44. Market clearing condition

$$Y_t = (1 - \gamma_c) (p_t^c)^{\eta_c} C_t + (1 - \gamma_{Inv}) (p_t^{Inv})^{\eta_{Imp}} [I_t + a(u_t) \bar{K}_t] + G_t \\ + (1 - \gamma_{Exp}) \left[ \gamma_{Exp} (p_t^{Exp, Imp})^{1-\eta_x} + 1 - \gamma_{Exp} \right]^{\frac{\eta_x}{1-\eta_x}} (\tilde{p}_t^{Exp})^{-\varepsilon_{Exp}} (p_t^x)^{-\eta_f} Y_t^*$$

45. Definition of GDP

$$GDP_t = Y_t - (1 - \gamma_{Inv}) (p_t^{Inv})^{\eta_{Imp}} a(u_t) \bar{K}_t$$

46. Relative price of imported consumption goods

$$\frac{p_t^{c, Imp}}{p_{t-1}^{c, Imp}} = \frac{\pi_t^{c, Imp}}{\pi_t^d}$$

47. Relative price of imported investment goods

$$\frac{p_t^{Inv, Imp}}{p_{t-1}^{Inv, Imp}} = \frac{\pi_t^{Inv, Imp}}{\pi_t^d}$$

48. Relative price of imported goods used in export sector

$$\frac{p_t^{Exp,Imp}}{p_{t-1}^{Exp,Imp}} = \frac{\pi_t^{Exp,Imp}}{\pi_t^d}$$

49. Relative price of exported goods

$$\frac{p_t^{Exp}}{p_{t-1}^{Exp}} = \frac{\pi_t^{Exp}}{\Delta E_t \pi_t^*}$$

50. Change of real exchange rate

$$\frac{q_t}{q_{t-1}} = \frac{\Delta E_t \pi_t^*}{\pi_t^c}$$

51. Relative price of final consumption goods

$$p_t^c = \left[ 1 - \gamma_c + \gamma_c (p_t^{c,Imp})^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}$$

52. Relative price of investment goods

$$p_t^{Inv} = \left[ 1 - \gamma_{Inv} + \gamma_{Inv} (p_t^{Inv,Imp})^{1-\eta_{Inv}} \right]^{\frac{1}{1-\eta_{Inv}}}$$

53. Inflation of consumption goods

$$\pi_t^c = \pi_t^d \left[ \frac{1 - \gamma_c + \gamma_c (p_t^{c,Imp})^{1-\eta_c}}{1 - \gamma_c + \gamma_c (p_{t-1}^{c,Imp})^{1-\eta_c}} \right]^{\frac{1}{1-\eta_c}}$$

54. Inflation of investment goods

$$\pi_t^{Inv} = \pi_t^d \left[ \frac{1 - \gamma_{Inv} + \gamma_{Inv} (p_t^{Inv,Imp})^{1-\eta_{Inv}}}{1 - \gamma_{Inv} + \gamma_{Inv} (p_{t-1}^{Inv,Imp})^{1-\eta_{Inv}}} \right]^{\frac{1}{1-\eta_{Inv}}}$$

55. First auxiliary variable of wage Phillips curve

$$X_{2,t}^w = \frac{\lambda_t}{\lambda_w} (\tilde{w}_t)^{\varepsilon_w} n_t + \beta \theta^w X_{2,t+1}^w \left( \frac{w_{t+1}}{w_t} \right) (\pi_{t+1}^w)^{\varepsilon_w - 1}$$

56. Second auxiliary variable of wage Phillips curve

$$X_{1,t}^w = \xi_t^n ((\tilde{w}_t)^{\varepsilon_w} n_t)^{1+\varphi} + \beta \theta^w X_{1,t+1}^w (\pi_{t+1}^w)^{\varepsilon_w(1+\varphi)}$$

57. Optimal ratio in labor market

$$\frac{X_{1,t}^w}{X_{2,t}^w} = w_t \left[ \frac{1 - \theta^w (\pi_t^w)^{1-\varepsilon_w}}{1 - \theta^w} \right]^{1 - \frac{\varepsilon_w}{\varepsilon_w - 1} (1+\varphi)}$$

58. Wage distortion in labor market

$$\tilde{w}_t = \left[ (1 - \theta^w) \left( \frac{1 - \theta^w (\pi_t^w)^{\varepsilon_w - 1}}{1 - \theta^w} \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} + \theta^w \left( \frac{\tilde{w}_{t-1}}{\pi_t^w} \right)^{-\varepsilon_w} \right]^{-\frac{1}{\varepsilon_w}}$$

59. Change of nominal wages

$$\pi_t^w = \frac{w_t \pi_t^d}{w_{t-1}}$$

60. Relationship between labor supply and labor input in production function

$$N_t = (\tilde{w}_t)^{\varepsilon_w} n_t$$

61. Import equation

$$Imp_t = q_t p_t^c \left( C_t^{Imp} (\tilde{p}_t^{c, Imp})^{-\varepsilon_{c, Imp}} + I_t^{Imp} (\tilde{p}_t^{Inv, Imp})^{-\varepsilon_{Inv, Imp}} + Exp_t^{Imp} (\tilde{p}_t^{Exp, Imp})^{-\varepsilon_{Exp, Imp}} \right)$$

62. Taylor rule

$$\frac{R_t}{R^{ss}} = \rho_R \frac{R_{t-1}}{R^{ss}} + (1 - \rho_R) \left[ \mu_\pi \frac{\pi_{t+1}^c}{\pi^{target}} + \mu_{gdp} \frac{GDP_{t+1}}{GDP^{ss}} \right] + \sigma_t^R$$

63. Price mark-up shock in production of domestic intermediate goods

$$\tau_t^d = \rho_{\tau^d} \tau_{t-1}^d + \sigma_t^{\tau^d}$$

64. Price mark-up shock for imported consumption goods

$$\tau_t^{c, Imp} = \rho_{\tau^{c, Imp}} \tau_{t-1}^{c, Imp} + \sigma_t^{\tau^{c, Imp}}$$

65. Price mark-up shock for imported investment goods

$$\tau_t^{Inv, Imp} = \rho_{\tau^{Inv, Imp}} \tau_{t-1}^{Inv, Imp} + \sigma_t^{\tau^{Inv, Imp}}$$



66. Price mark-up shock for imported goods used in export sector

$$\tau_t^{Exp,Imp} = \rho_{\tau^{Exp,Imp}} \tau_{t-1}^{Exp,Imp} + \sigma_t^{\tau^{Exp,Imp}}$$

67. Price mark-up shock of exported goods

$$\tau_t^{Exp} = \rho_{\tau^{Exp}} \tau_{t-1}^{Exp} + \sigma_t^{\tau^{Exp}}$$

68. AR (1) process for productivity

$$Z_t = \rho_Z Z_{t-1} + \sigma_t^Z$$

69. Consumption preference shock

$$\xi_t^c = \rho_{\xi^c} \xi_{t-1}^c + \sigma_t^{\xi^c}$$

70. Disutility from labor supply

$$\xi_t^n = \rho_{\xi^n} \xi_{t-1}^n + \sigma_t^{\xi^n}$$

71. Government spending

$$G_t = \rho_G G_{t-1} + (1 - \rho_G) G^{ss} + \sigma_t^G$$

72. Marginal efficiency of investment

$$\Psi_t = \rho_{\Psi} \Psi_{t-1} + \sigma_t^{\Psi}$$

73. Risk premium shock

$$\Omega_t = \rho_{\Omega} \Omega_{t-1} + \sigma_t^{\Omega}$$

74. Foreign inflation

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_t^{\pi^*}$$

75. Foreign demand

$$Y_t^* = \rho_{Y^*} Y_{t-1}^* + (1 - \rho_{Y^*}) Y^{*,ss} + \sigma_t^{Y^*}$$

76. Foreign interest rate

$$R_t^* = \rho_{R^*} R_{t-1}^* + (1 - \rho_{R^*}) R^{*,ss} + \sigma_t^{R^*}$$

### Appendix 3. First Order Approximation of Investment Adjustment Cost Function

This paper discusses investment adjustment cost function of the form:

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = 0.5 \left\{ e^{\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} + e^{-\sqrt{S''}\left(\frac{I_t}{I_{t-1}} - 1\right)} - 2 \right\}$$

First order approximation of the  $f(I_t, I_{t-1})$  function is represented by the following:

$$f(I_t, I_{t-1}) = f(I^{ss}, I^{ss}) + f_{I_t}(I^{ss}, I^{ss})(I_t - I^{ss}) + f_{I_{t-1}}(I^{ss}, I^{ss})(I_{t-1} - I^{ss}) + [O^2]$$

where  $f(I^{ss}, I^{ss})$  is the value of function in steady state,  $f_{I_t}(I^{ss}, I^{ss})$  is the first order derivative of function  $f(I_t, I_{t-1})$  with respect to  $I_t$  at  $I_t = I^{ss}$  and  $I_{t-1} = I^{ss}$ .  $f_{I_{t-1}}(I^{ss}, I^{ss})$  is the derivative of  $f(I_t, I_{t-1})$  with respect to  $I_{t-1}$ .  $[O^2]$  includes second and higher order terms, which we assume to be zero in the first order approximation.

We use the above formula for first order Taylor approximation to the investment adjustment cost function.

$$\begin{aligned} \Phi\left(\frac{I_t}{I_{t-1}}\right) &= 0.5e^{\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} + 0.5e^{-\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} - 1 + 0.5e^{\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} + \\ &+ 0.5e^{\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} \frac{\sqrt{S''}}{I^{ss}} (I_t - I^{ss}) + 0.5e^{\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} \left(-\frac{\sqrt{S''}I^{ss}}{(I^{ss})^2}\right) (I_{t-1} - I^{ss}) + \\ &+ 0.5e^{-\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} \left(-\frac{\sqrt{S''}}{I^{ss}}\right) (I_t - I^{ss}) + 0.5e^{-\sqrt{S''}\left(\frac{I^{ss}}{I^{ss}} - 1\right)} \left(\frac{\sqrt{S''}I^{ss}}{(I^{ss})^2}\right) (I_{t-1} - I^{ss}) \end{aligned}$$

Some simplification yields to the following:

$$\begin{aligned} \Phi\left(\frac{I_t}{I_{t-1}}\right) &= 0.5e^0 + 0.5e^0 - 1 + 0.5e^0\sqrt{S''}\left(\frac{I_t - I^{ss}}{I^{ss}}\right) - 0.5e^0\sqrt{S''}\left(\frac{I_{t-1} - I^{ss}}{I^{ss}}\right) - \\ &- 0.5e^0\sqrt{S''}\left(\frac{I_t - I^{ss}}{I^{ss}}\right) + 0.5e^0\sqrt{S''}\left(\frac{I_{t-1} - I^{ss}}{I^{ss}}\right) \end{aligned}$$

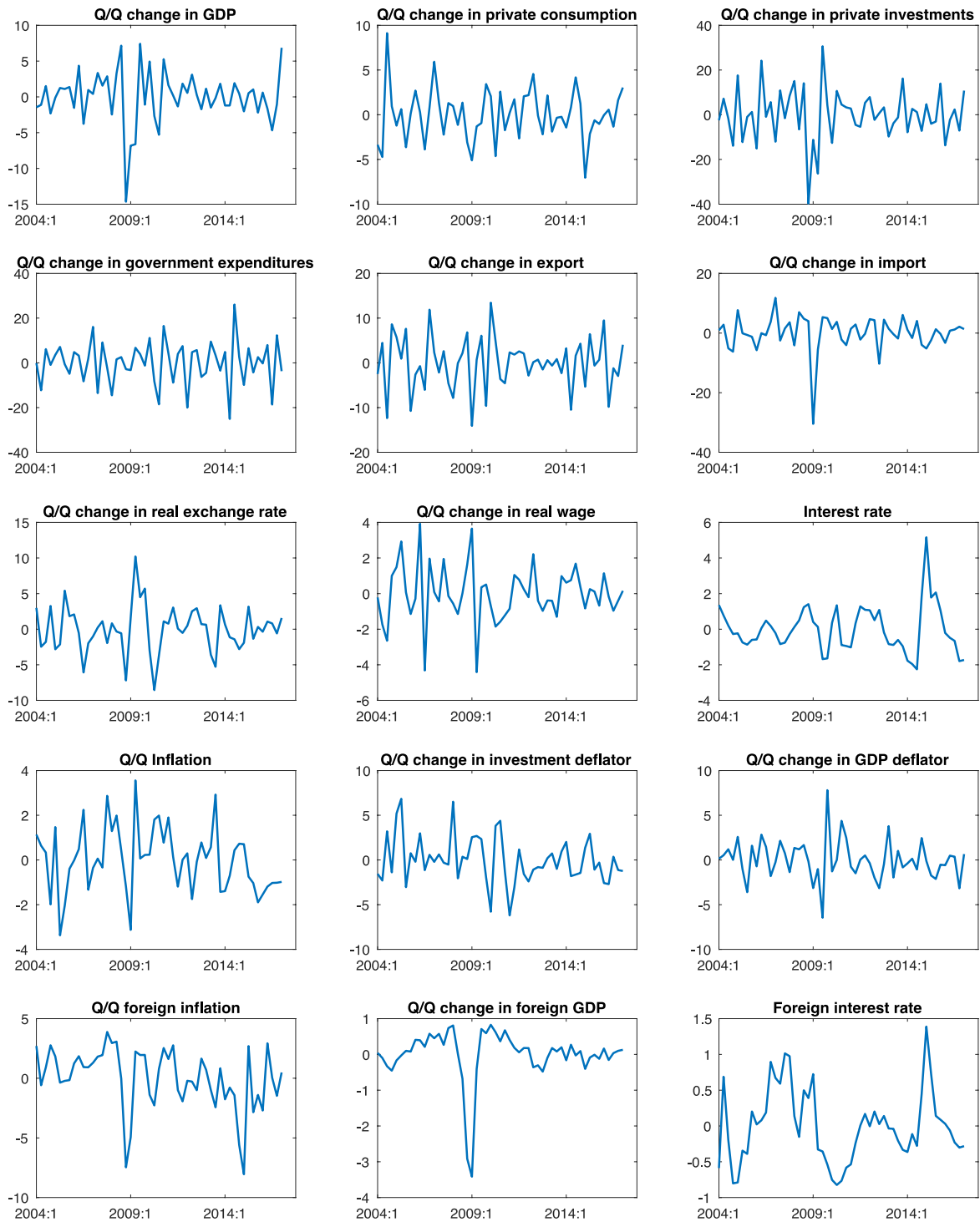
Further simplification gives:

$$\begin{aligned} \Phi\left(\frac{I_t}{I_{t-1}}\right) &= 0.5 + 0.5 - 1 + 0.5\sqrt{S''}\left(\frac{I_t - I^{ss}}{I^{ss}}\right) - 0.5\sqrt{S''}\left(\frac{I_t - I^{ss}}{I^{ss}}\right) + \\ &+ 0.5\sqrt{S''}\left(\frac{I_{t-1} - I^{ss}}{I^{ss}}\right) - 0.5\sqrt{S''}\left(\frac{I_{t-1} - I^{ss}}{I^{ss}}\right) \end{aligned}$$

All terms cancel each other, and we have zero value of investment adjustment costs by applying the first order approximation.

$$\Phi\left(\frac{I_t}{I_{t-1}}\right) = 0.$$

## Appendix 4. Data Used in the Estimation of Theoretical Model



## Appendix 5. Calibration and Estimation Results

Table 5.1. Calibrated parameters

Parameters	Description	Values
$\beta$	Discount factor	0.99
$\gamma_c$	Share of imported goods in consumption	0.35
$\gamma_{Inv}$	Share of imported goods in investment	0.32
$\gamma_{Exp}$	Share of imported inputs in export	0.35
$\varepsilon_d$	Elasticity of substitution between varieties of domestically produced goods	6.0
$\varepsilon_{c,Imp}$	Elasticity of substitution between varieties of imported consumption goods	6.0
$\varepsilon_{Inv,Imp}$	Elasticity of substitution between varieties of imported investment good	6.0
$\varepsilon_{Exp,Imp}$	Elasticity of substitution between varieties of imported goods used in export sector	6.0
$\varepsilon_{Exp}$	Elasticity of substitution between varieties of exported goods	6.0
$\alpha$	Share of capital in production function	0.55
$G^{ss}$	Share of government expenditures in GDP	0.17
$\pi^{target}$	Inflation target	1.00

Table 5.2. Prior and posterior distributions of structural parameters

	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
$\theta^d$	Price stickiness coefficient of home produced goods	0.75	0.90	0.87	0.93	Beta	0.075
$\theta^{c,Imp}$	Price stickiness coefficient of imported consumption	0.75	0.60	0.50	0.70	Beta	0.075
$\theta^{Inv,Imp}$	Price stickiness coefficient of imported investment goods	0.75	0.58	0.47	0.69	Beta	0.075
$\theta^{Exp,Imp}$	Price stickiness coefficient of imported goods used in export sector	0.75	0.49	0.40	0.59	Beta	0.075
$\theta^{Exp}$	Price stickiness coefficient of exported goods	0.75	0.73	0.64	0.87	Beta	0.075
$\theta^w$	Wage stickiness coefficient	0.75	0.84	0.7	0.93	Beta	0.075
$\eta_c$	Elasticity of substitution between domestic and imported consumption goods	3.0	1.26	0.99	1.50	Gamma	0.45
$\eta_{Inv}$	Elasticity of substitution between domestic and imported investment goods	3.0	1.16	0.95	1.32	Gamma	0.45
$\eta_f$	Elasticity of export	3.0	0.77	0.49	1.06	Gamma	0.45
$\eta_x$	Elasticity of substitution between domestic and imported inputs in export sector	3.0	1.40	0.99	1.79	Gamma	0.45
$\varphi$	Inverse elasticity of labor supply	2.0	1.88	1.41	2.34	Gamma	0.3
$hab$	Habit parameter	0.5	0.63	0.54	0.73	Beta	0.2
$S''$	Investment adjustment costs parameter	9.0	5.25	1.78	8.48	Gamma	3.1
$\varrho^a$	Parameter in capital utilization	0.2	0.16	0.07	0.25	Gamma	0.075
$\lambda_w = \frac{\varepsilon_w}{\varepsilon_w - 1}$	Mark-up on wages	1.3	1.16	1.01	1.32	Gamma	0.15
$\rho_R$	Persistence coefficient in Taylor rule	0.7	0.75	0.68	0.82	Beta	0.12
$\mu_\pi$	Reaction of interest rate to inflation expectations in Taylor rule	1.5	1.56	1.19	1.94	Gamma	0.25

$\mu_{gdp}$	Reaction of interest rate to the output deviation from its steady state	0.25	0.14	0.09	0.18	Gamma	0.05
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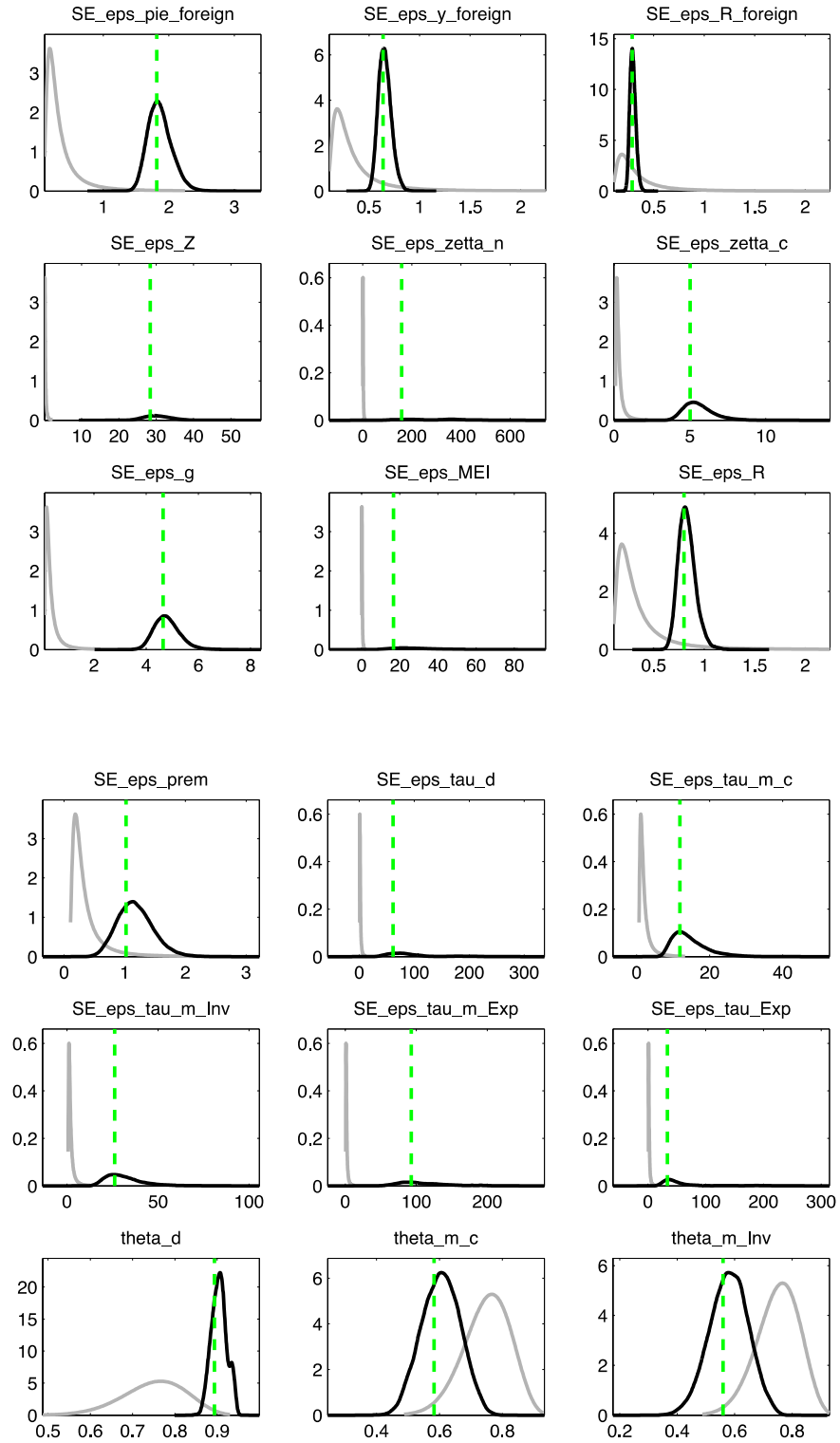
Table 5.3. Prior and posterior distribution of shocks' autoregressive parameters

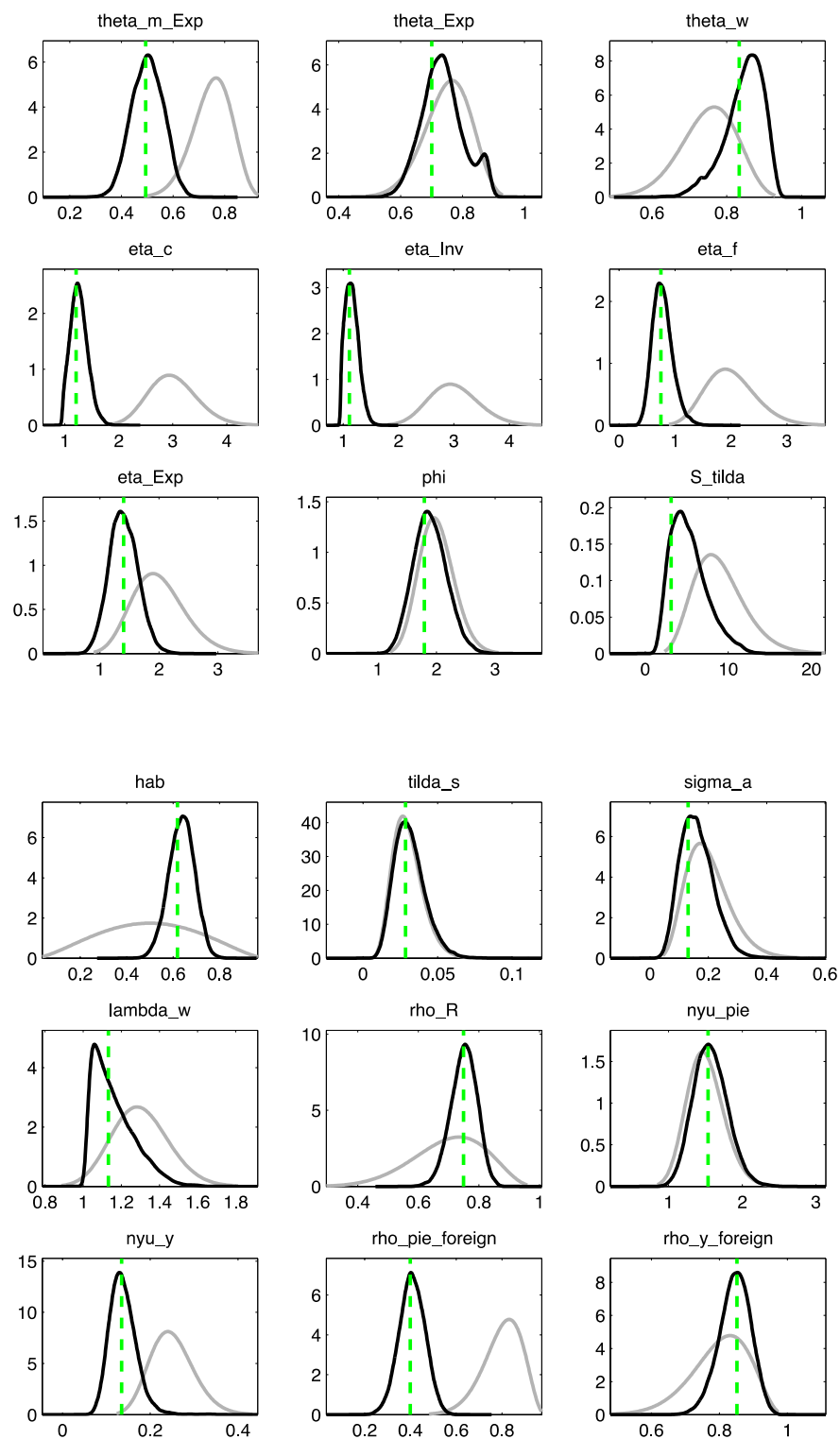
	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
$\rho_{\xi^c}$	Consumption preference	0.80	0.64	0.52	0.76	Beta	0.085
$\rho_{\xi^n}$	Labor supply	0.80	0.68	0.52	0.83	Beta	0.085
$\rho_Z$	Productivity	0.80	0.51	0.41	0.61	Beta	0.085
$\rho_{\Psi}$	Marginal efficiency of investment	0.80	0.47	0.35	0.58	Beta	0.085
$\rho_G$	Government spending	0.80	0.62	0.48	0.75	Beta	0.085
$\rho_{\Omega}$	Risk premium	0.80	0.72	0.62	0.84	Beta	0.085
$\rho_{\tau^d}$	Price mark-up of domestic intermediate goods	0.80	0.56	0.42	0.69	Beta	0.085
$\rho_{\tau^{c,Imp}}$	Price mark-up of imported consumption goods	0.80	0.59	0.46	0.74	Beta	0.085
$\rho_{\tau^{Inv,Imp}}$	Price mark-up of imported investment goods	0.80	0.58	0.46	0.71	Beta	0.085
$\rho_{\tau^{Exp,Imp}}$	Price mark-up of imported goods used in export sector	0.80	0.55	0.41	0.67	Beta	0.085
$\rho_{\tau^{Exp}}$	Price mark-up of exported goods	0.80	0.63	0.48	0.77	Beta	0.085
$\rho_{Y^*}$	Foreign demand	0.80	0.84	0.77	0.92	Beta	0.085
$\rho_{\pi^*}$	Foreign inflation	0.80	0.41	0.31	0.49	Beta	0.085
$\rho_{R^*}$	Foreign interest rate	0.80	0.74	0.66	0.82	Beta	0.085

Table 5.4. Prior and posterior distribution of shocks' standard errors

	Description	Prior mean	Posterior mean	5%	95%	Prior dist.	Prior SD
$\sigma^{\xi^c}$	Consumption preference	0.4	5.57	4.07	7.07	Inv. gamma	5.0
$\sigma^{\xi^n}$	Labor supply	2.4	288.03	95.13	456.90	Inv. Gamma	10.0
$\sigma^Z$	Productivity	0.4	30.30	24.68	35.81	Inv. Gamma	5.0
$\sigma^{\Psi}$	Marginal efficiency of Investment	0.4	27.41	10.45	43.73	Inv. Gamma	5.0
$\sigma^G$	Government spending	0.4	4.79	4.01	5.54	Inv. Gamma	5.0
$\sigma^{\Omega}$	Risk premium	0.4	1.17	0.69	1.63	Inv. Gamma	5.0
$\sigma^R$	Monetary policy	0.4	0.82	0.69	0.96	Inv. Gamma	5.0
$\sigma^{\tau^d}$	Price mark-up of domestic intermediate goods	2.4	90.67	35.27	171.36	Inv. Gamma	10.0
$\sigma^{\tau^{c,Imp}}$	Price mark-up of imported consumption goods	2.4	14.40	7.47	21.17	Inv. Gamma	10.0
$\sigma^{\tau^{Inv,Imp}}$	Price mark-up of imported investment goods	2.4	31.15	16.24	45.63	Inv. Gamma	10.0
$\sigma^{\tau^{Exp,Imp}}$	Price mark-up of imported goods used in export sector	2.4	107.11	58.04	158.52	Inv. Gamma	10.0
$\sigma^{\tau^{Exp}}$	Price mark-up of exported goods	2.4	55.85	17.05	103.44	Inv. Gamma	10.0
$\sigma^{Y^*}$	Foreign demand	0.4	0.65	0.55	0.75	Inv. Gamma	5.0
$\sigma^{\pi^*}$	Foreign inflation	0.4	1.86	1.57	2.15	Inv. Gamma	5.0
$\sigma^{R^*}$	Foreign interest rate	0.4	0.29	0.24	0.34	Inv. Gamma	5.0

Figure 5.1. Prior and posterior distributions





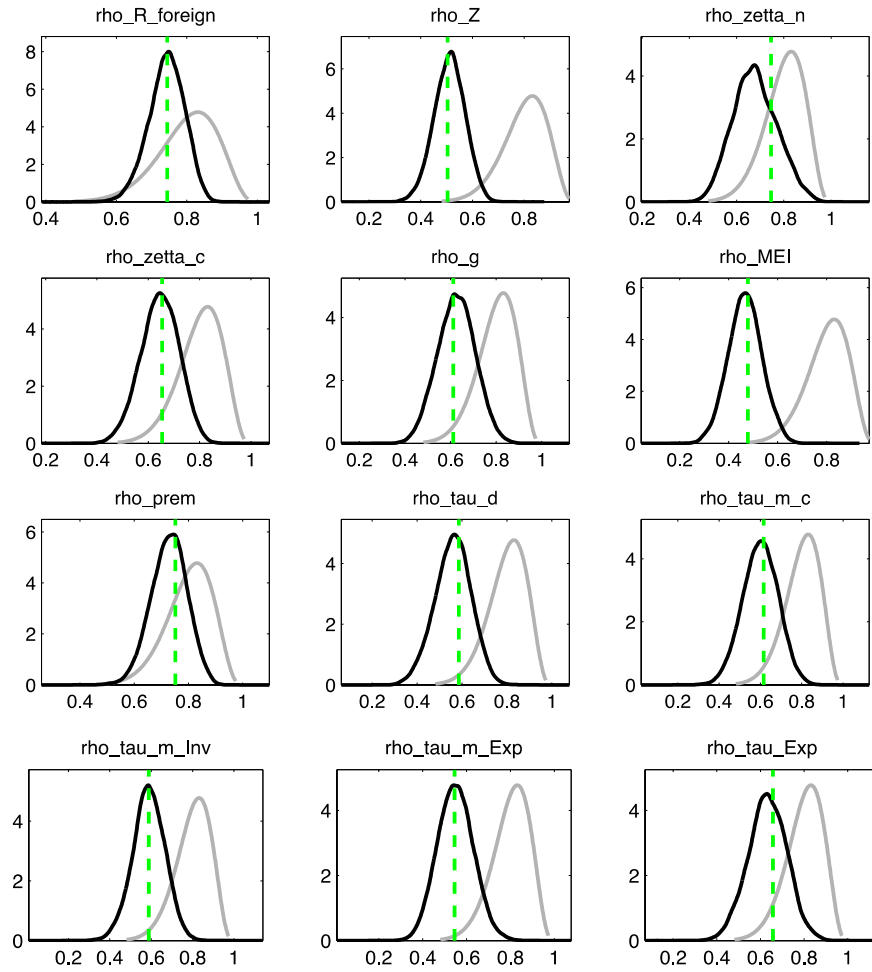
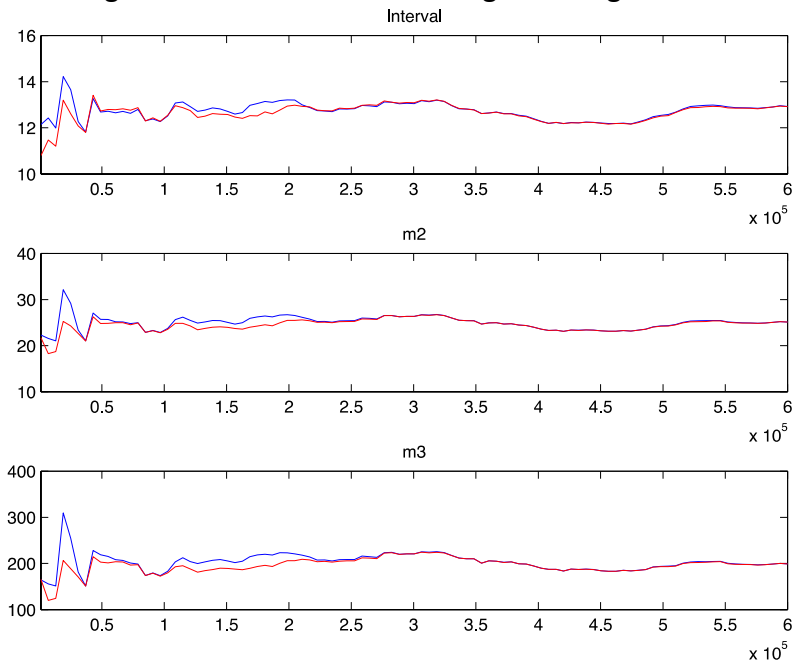


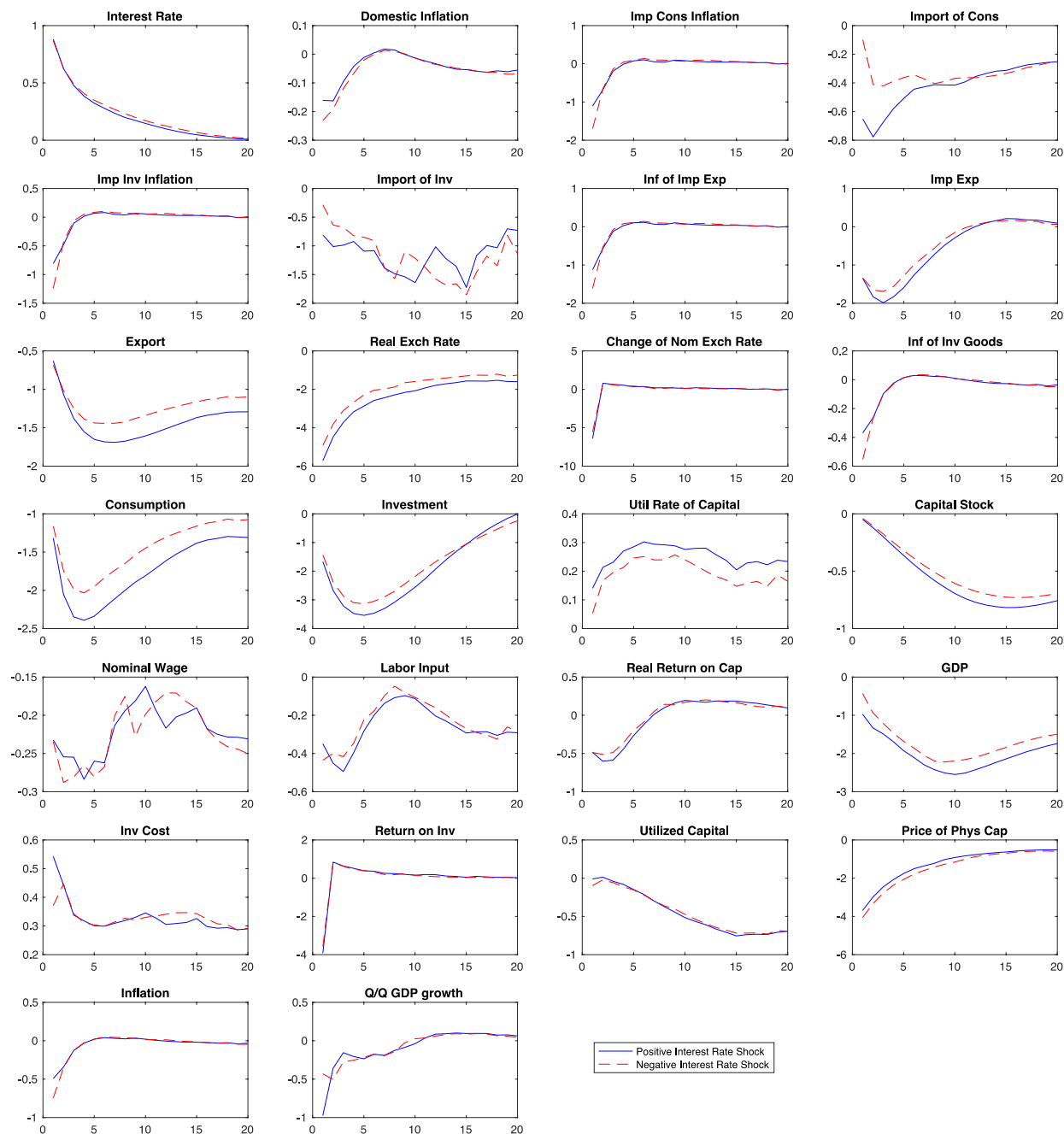
Figure 5.2. Multivariate convergence diagnostics





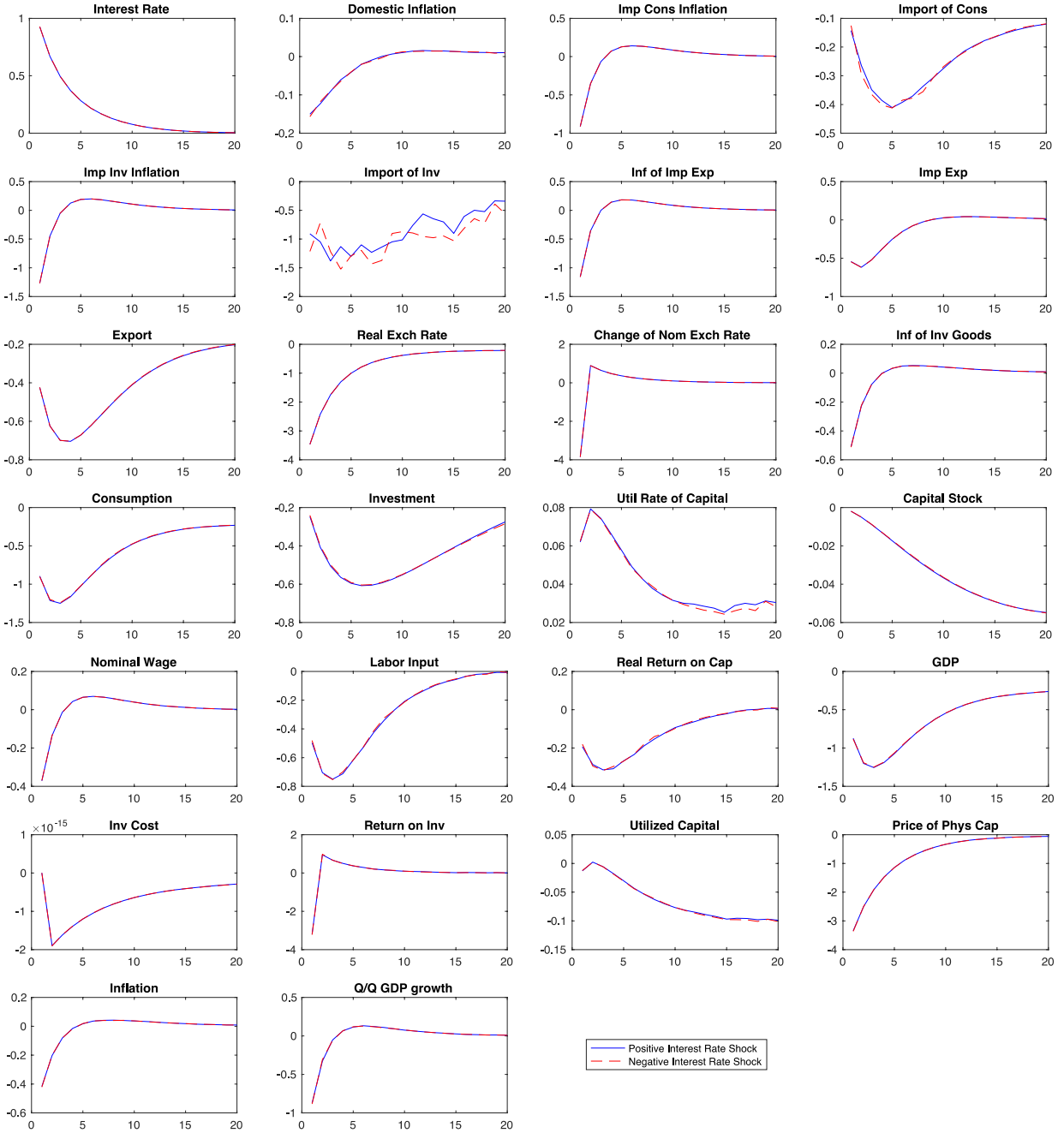
## Appendix 6. Impulse Response Functions for Different Cases

Figure 6.1. Monetary policy shock in the second order approximated model (Case 1)



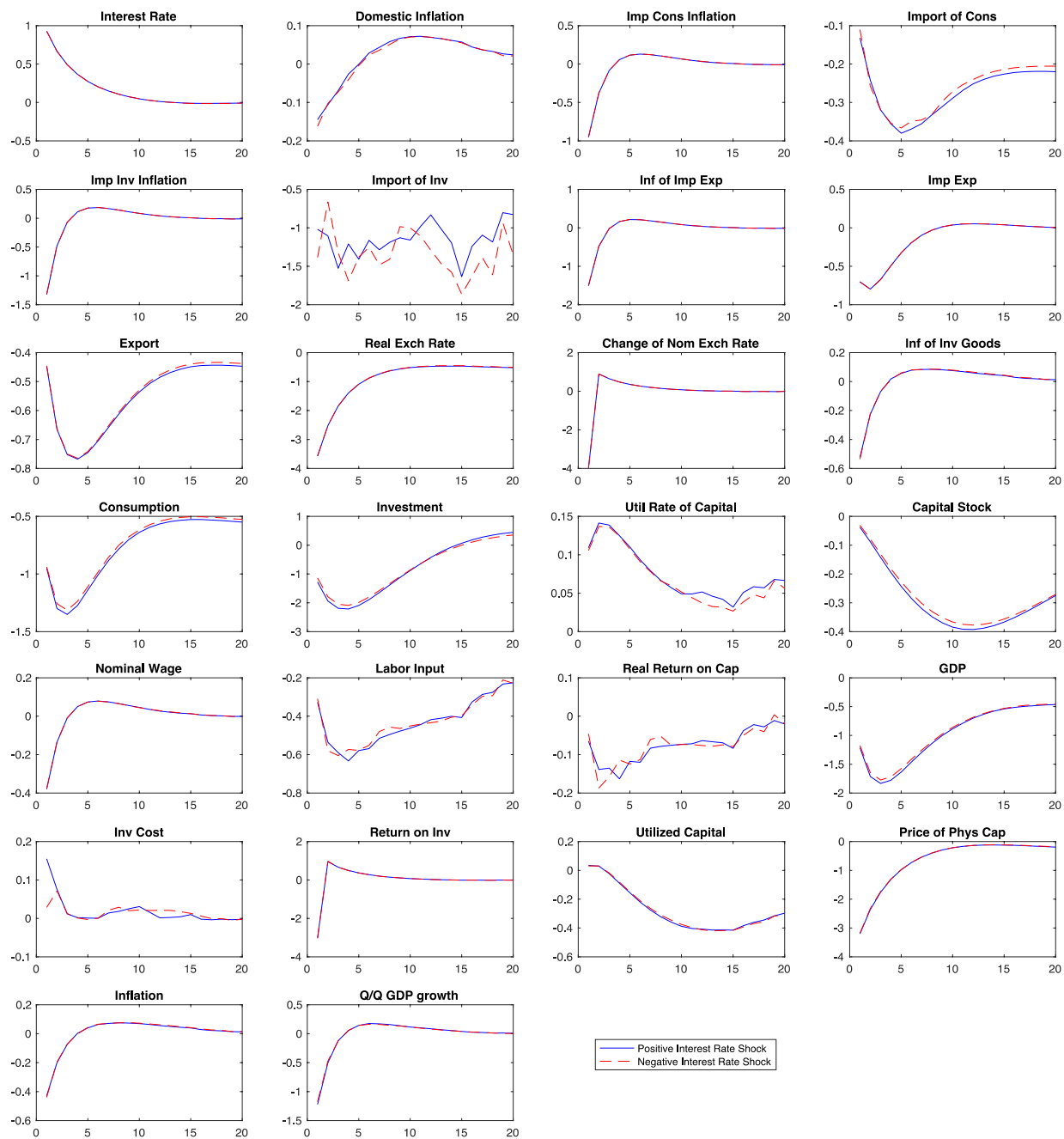
Generalized impulse response functions at ergodic mean based on the average of 200000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

Figure 6.2. Monetary policy shock in the log-linearized model. Standard assumptions are kept non-linear (Case 2)



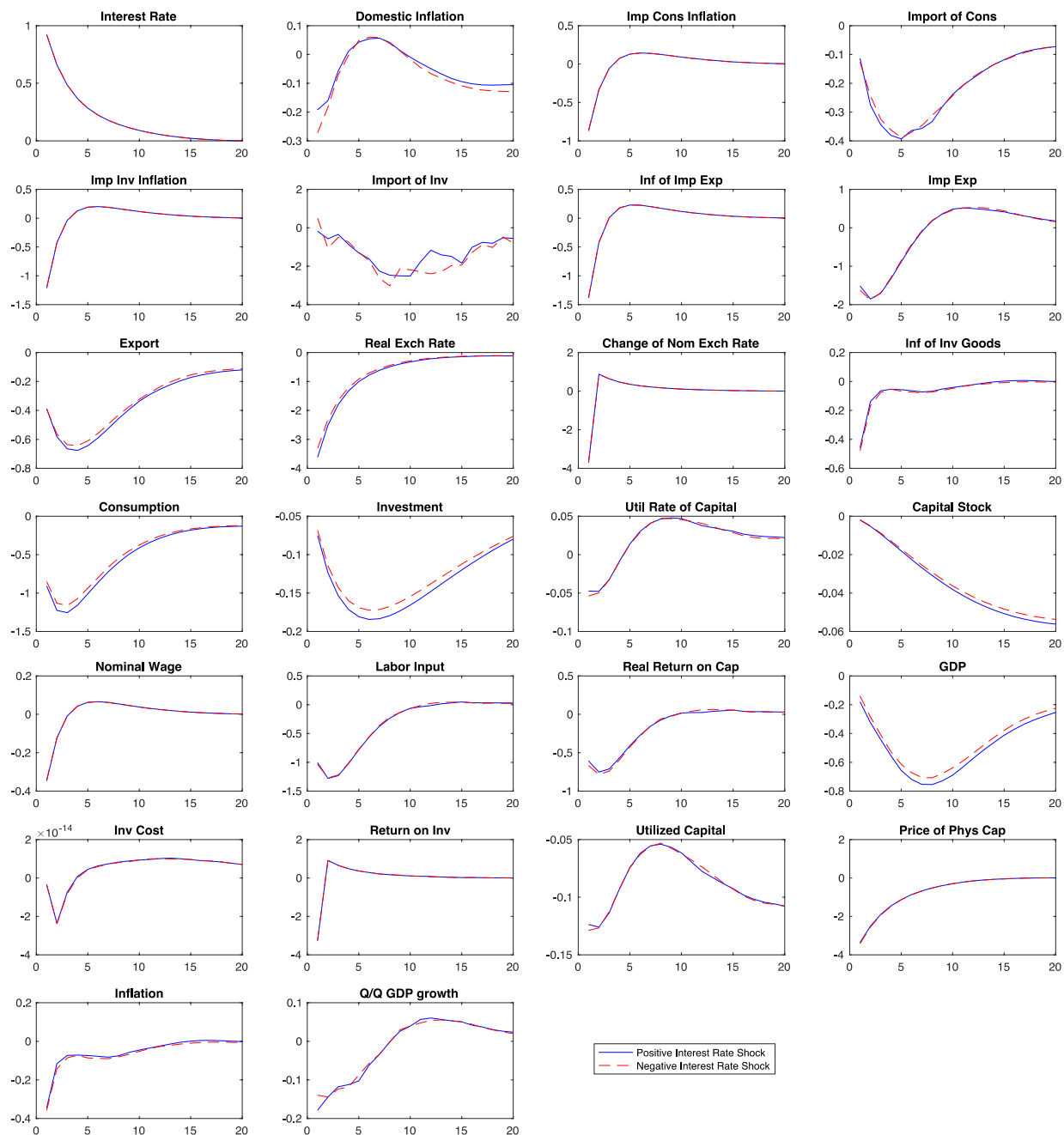
Generalized impulse response functions at ergodic mean based on the average of 200000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

Figure 6.3. Monetary policy shock in the model with second order approximated capital market frictions (Case 3)



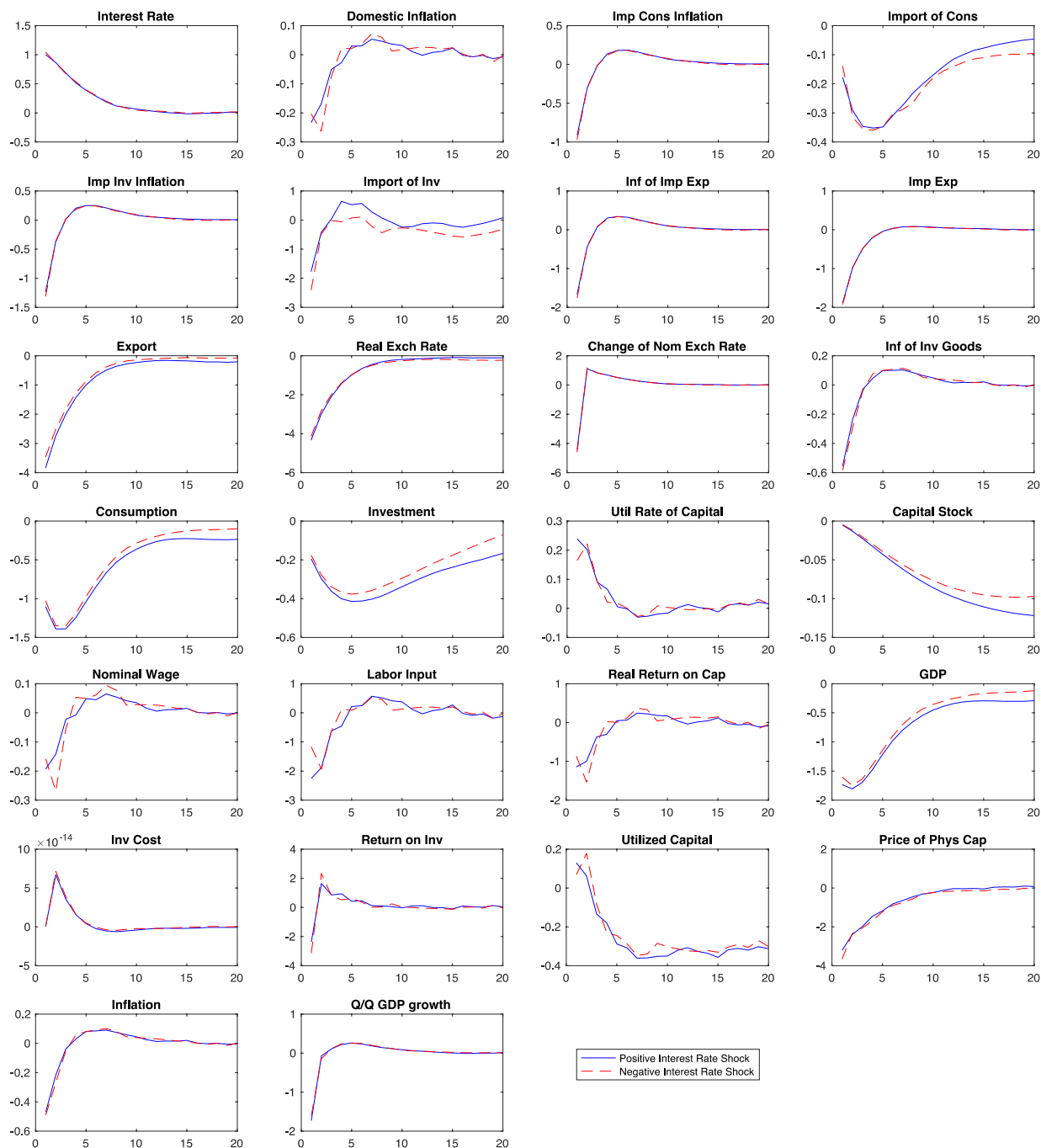
Generalized impulse response functions at ergodic mean based on the average of 20000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

Figure 6.4. Monetary policy shock in the model with second order approximated internal economy's Phillips curves (Case 4)



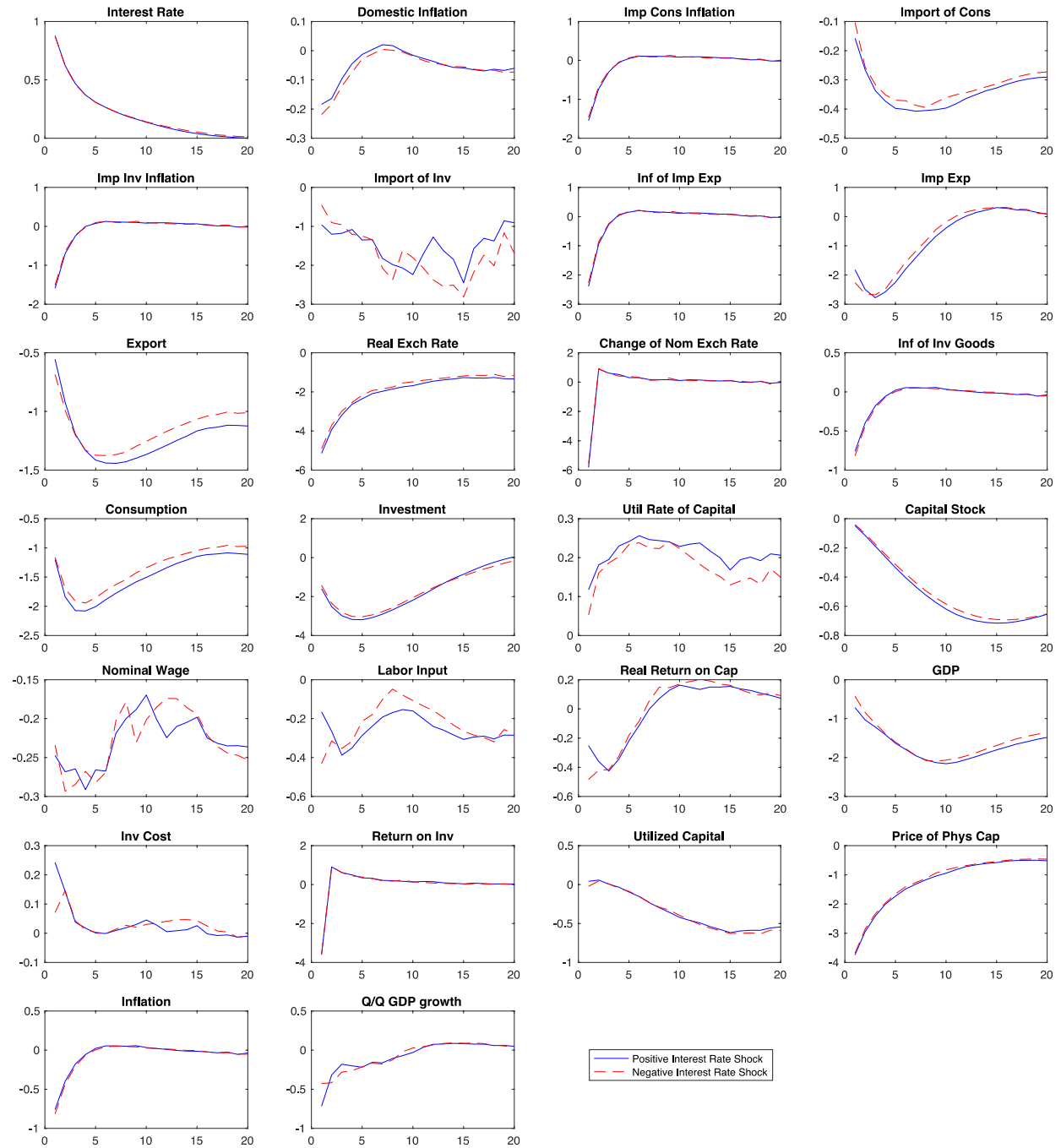
Generalized impulse response functions at ergodic mean based on the average of 200000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

Figure 6.5. Monetary policy shock in the model with second order approximated labor market frictions (Case 5)



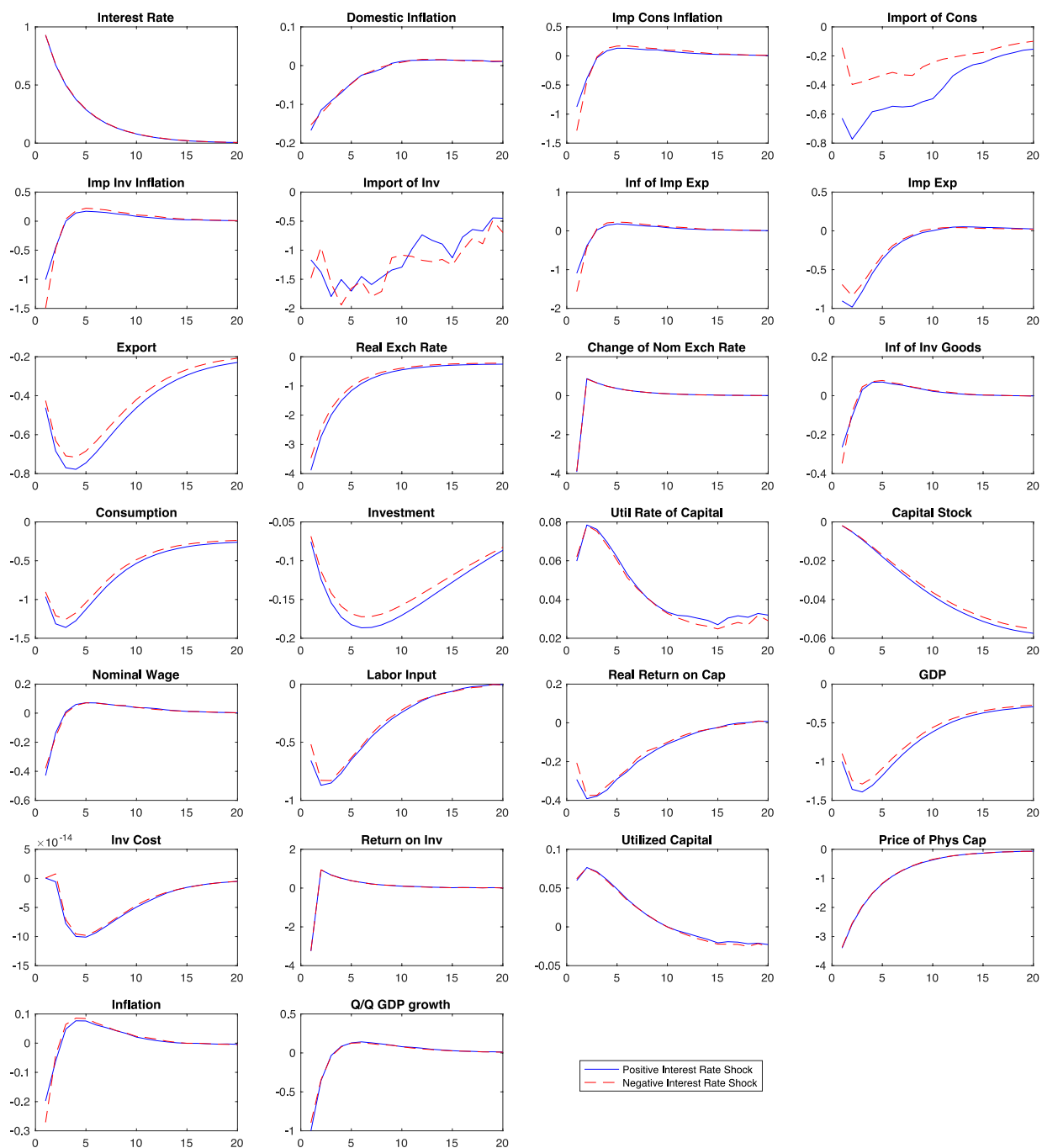
Generalized impulse response functions at ergodic mean based on the average of 200000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

Figure 6.6. Monetary policy shock in the nonlinear internal economy and log-linearized external sector model (Case 6)



Generalized impulse response functions at ergodic mean based on the average of 20000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

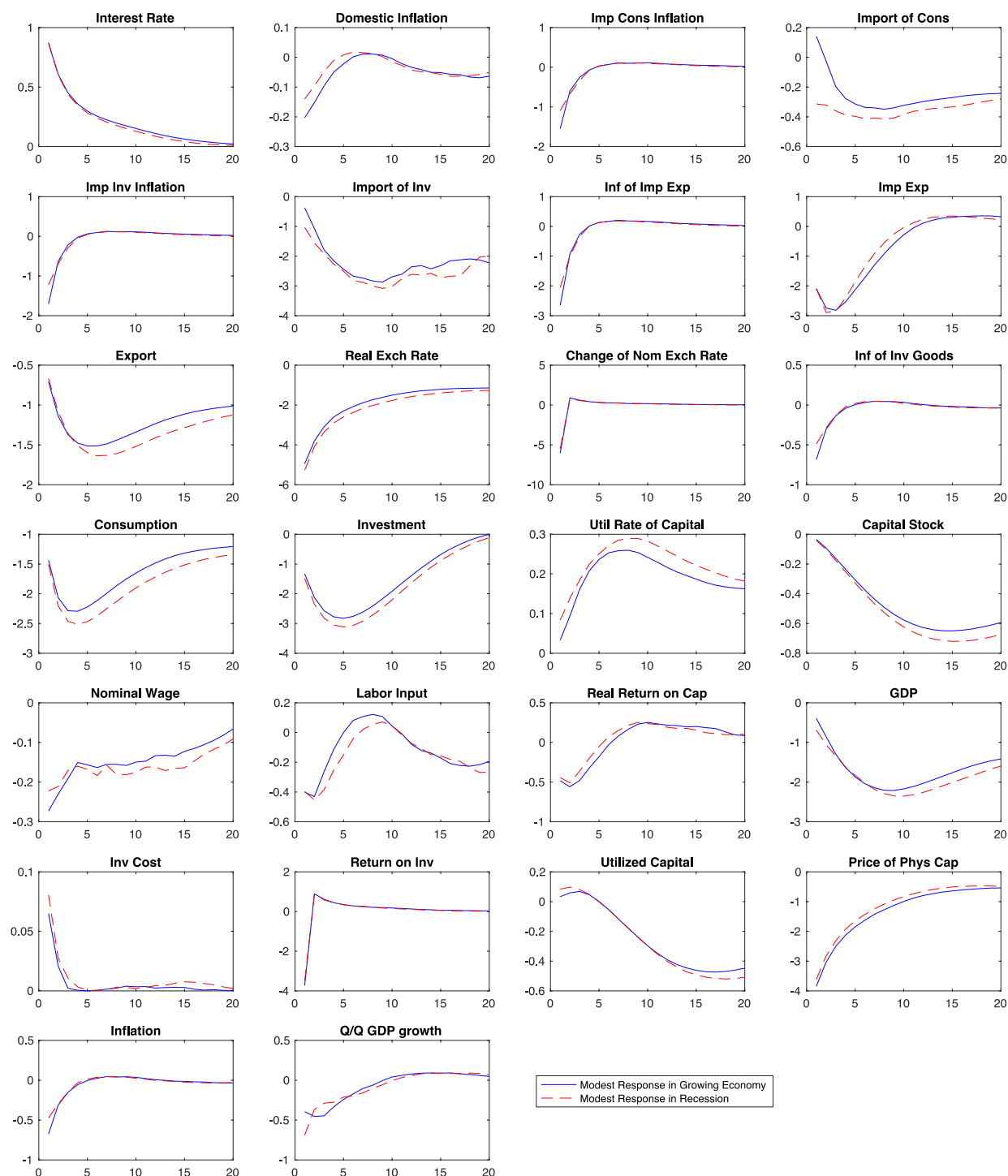
Figure 6.7. Monetary policy shock in the log-linearized internal economy and second order approximated external sector model (Case 7)



Generalized impulse response functions at ergodic mean based on the average of 200000 simulations. The responses to negative shock are shown as mirror images to facilitate the comparison

## Appendix 7. Policy Experiments in Growing Economies and Recessions

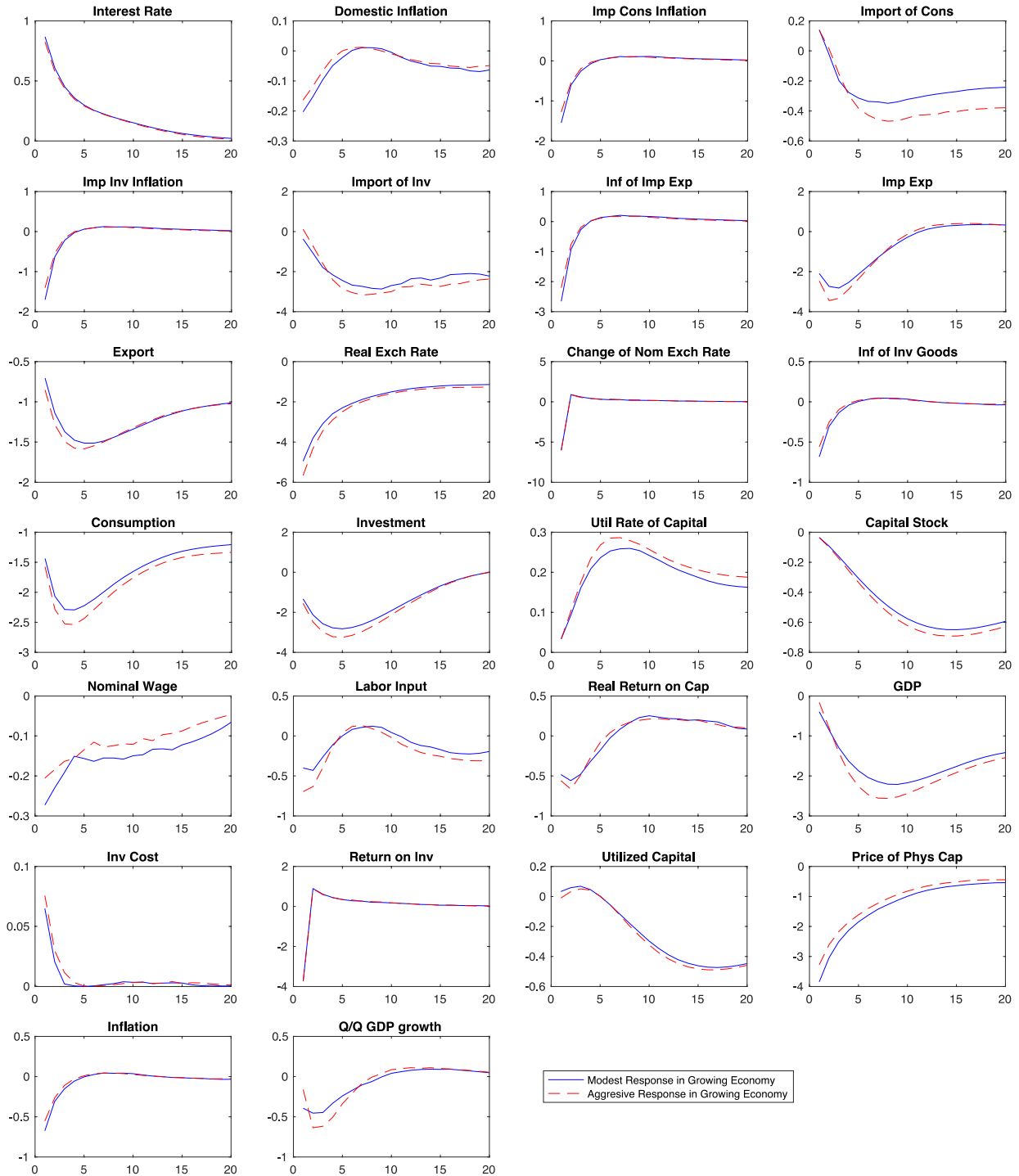
Figure 7.1. The response of the economy to modest monetary policy shock in demand driven growing economy and recession



Generalized impulse response functions when GDP is 5% above and 5% below steady state based on the average of 200000 simulations. The recession graphs are shown as mirror images to facilitate the comparison

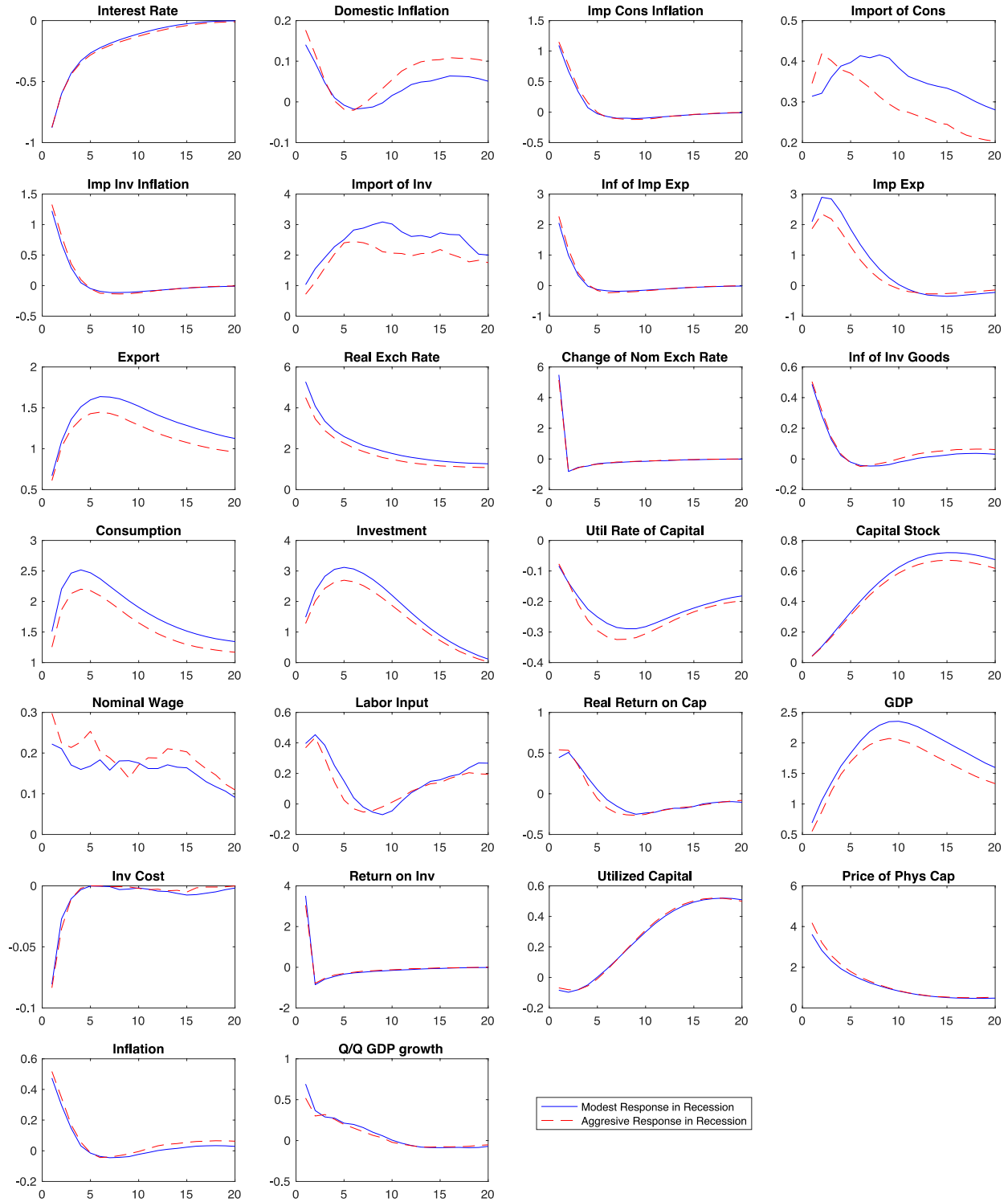


Figure 7.2. The response of the economy to modest and aggressive monetary policy shocks in demand driven growing economy



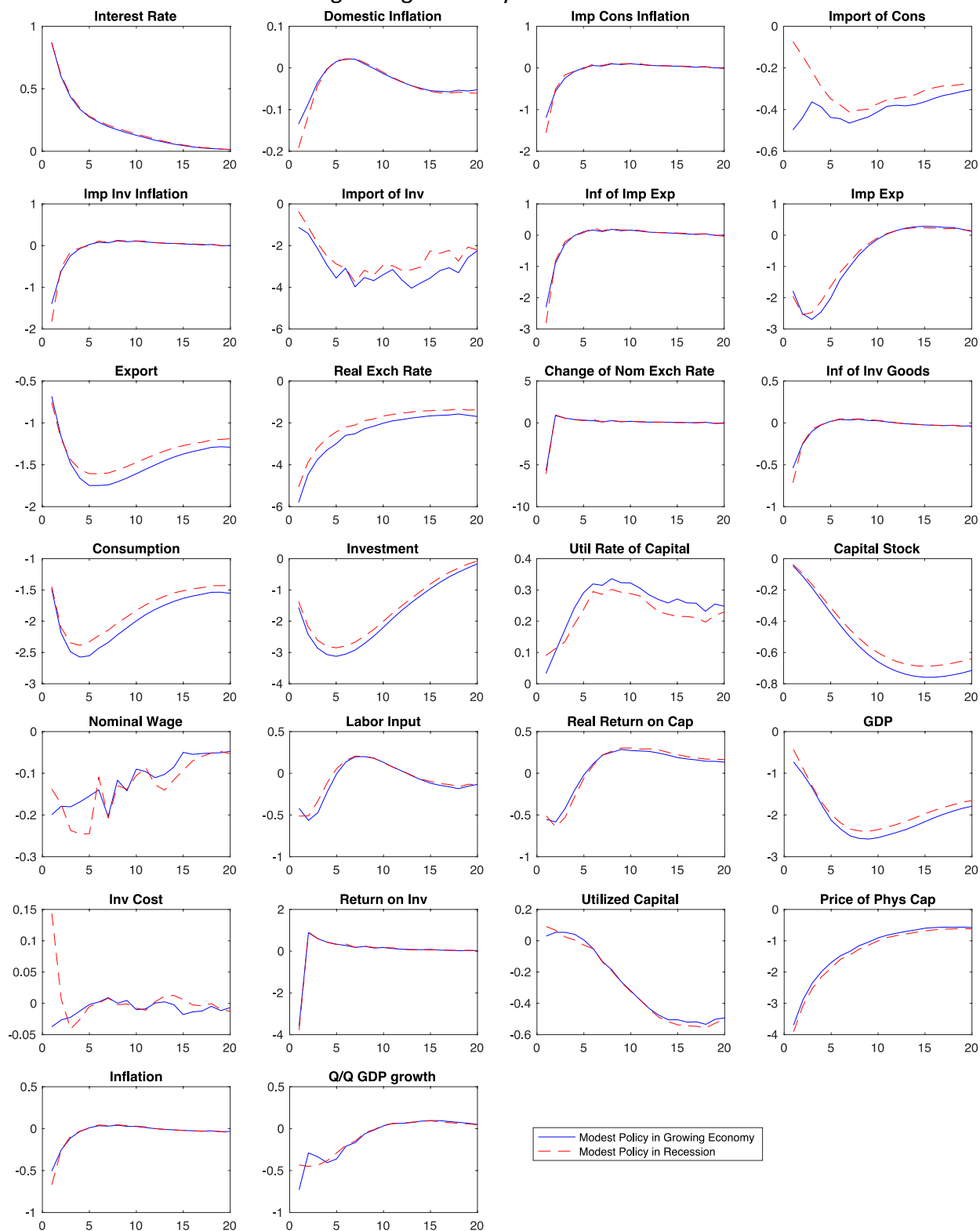
Generalized impulse response functions when GDP is 5% above steady state based on the average of 200000 simulations. The aggressive response is rescaled to facilitate the comparison

Figure 7.3. The response of the economy to modest and aggressive monetary policy shocks in demand driven recession



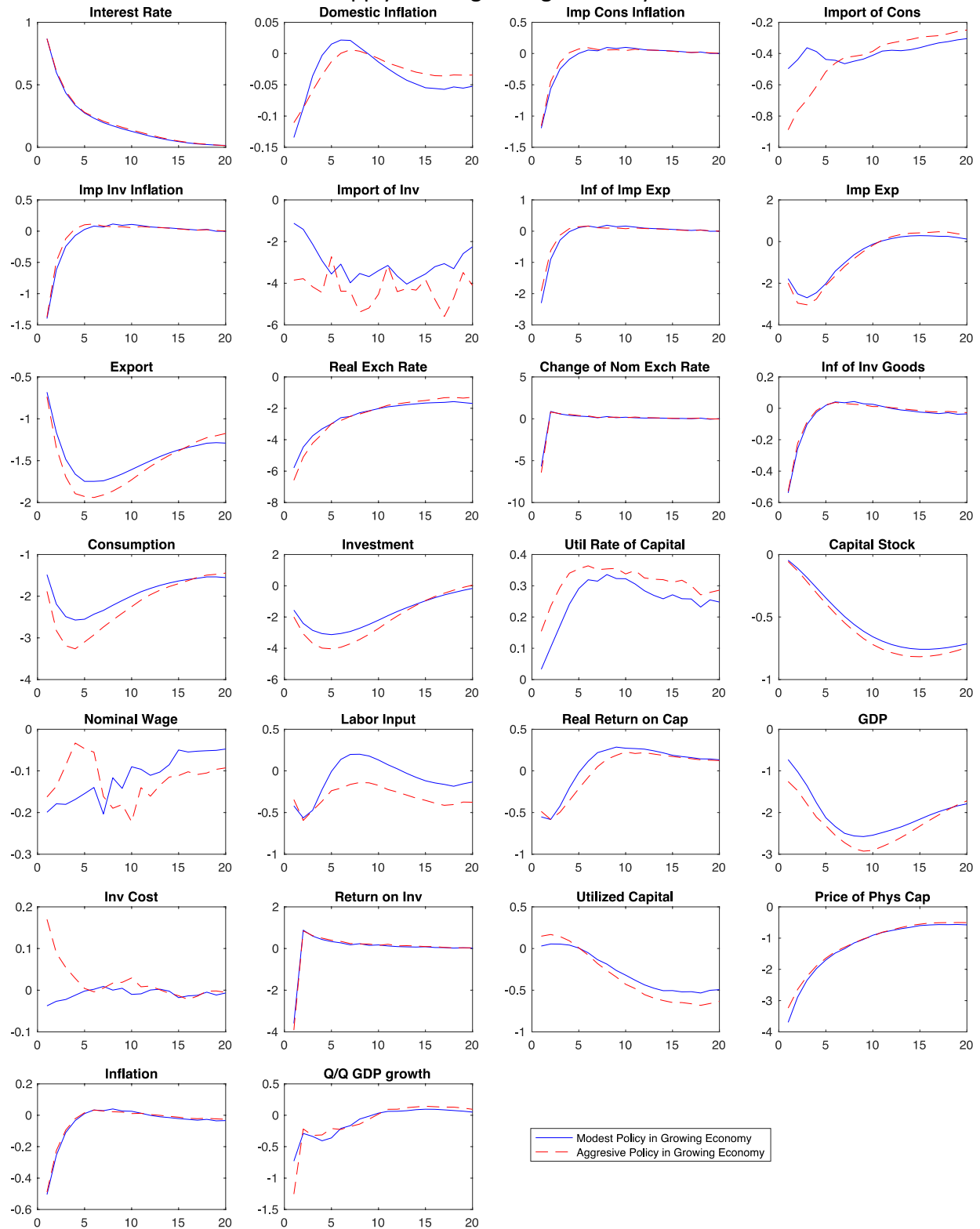
Generalized impulse response functions when GDP is 5% below steady state based on the average of 200000 simulations. The aggressive response is rescaled to facilitate the comparison

Figure 7.4. The response of the economy to modest monetary policy shock in supply driven growing economy and recession



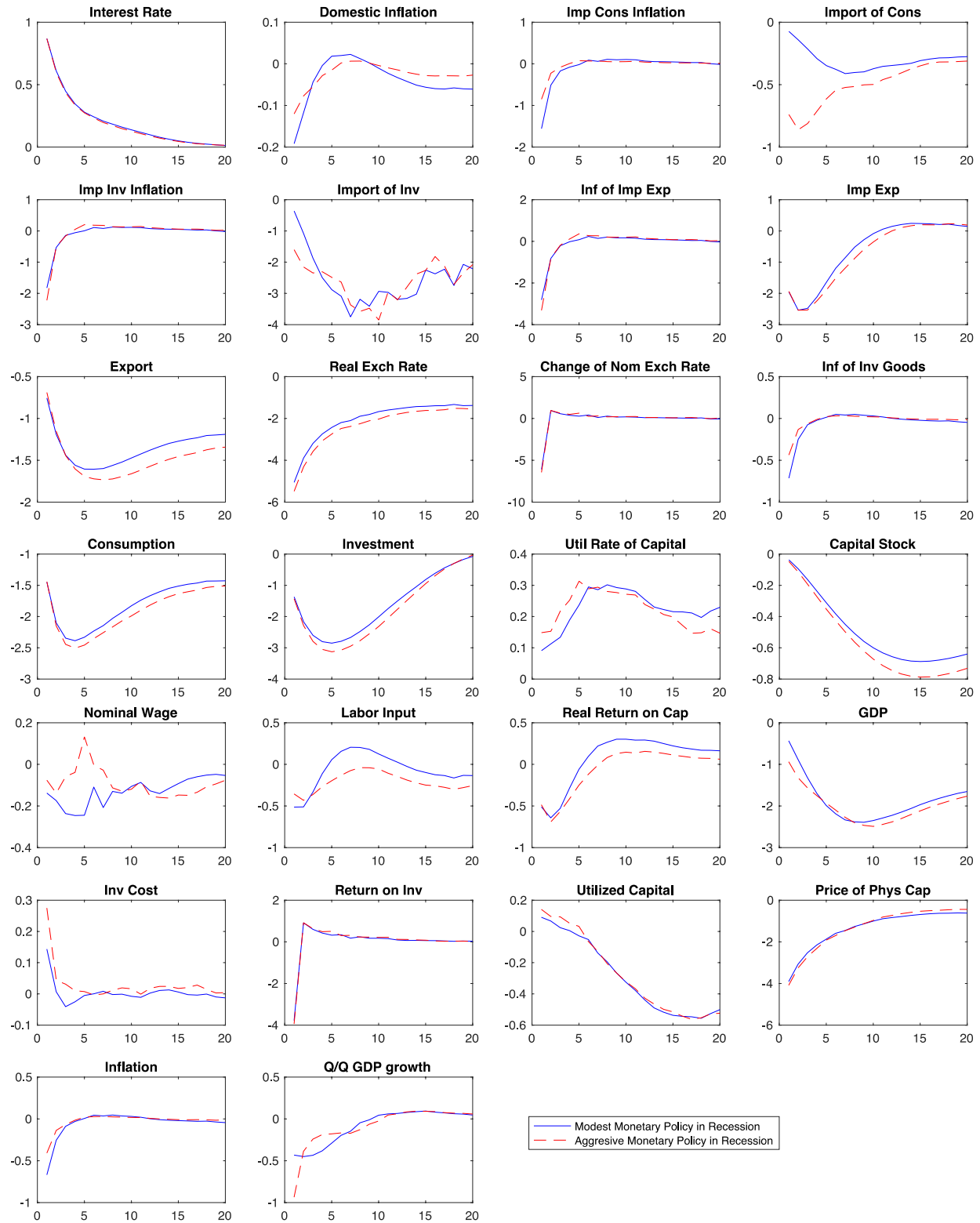
Generalized impulse response functions when GDP is 5% above and below steady state based on the average of 200000 simulations. The recession graphs are shown as mirror images to facilitate the comparison

Figure 7.5. The response of the economy to modest and aggressive monetary policy shocks in supply driven growing economy



Generalized impulse response functions when GDP is 5% above steady state based on the average of 200000 simulations. The aggressive response is rescaled to facilitate the comparison

Figure 7.6. The response of the economy to modest and aggressive monetary policy shocks in supply driven recession



Generalized impulse response functions when GDP is 5% below steady state based on the average of 200000 simulations. The aggressive response is rescaled to facilitate the comparison

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