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Forecasting the Volatility of
Philippine Inflation using
GARCH Models

Haydee L. Ramon

Series No. 2008-01

January 2008

Center for Monetary and Financial Policy
Monetary Policy Sub-Sector



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ABSTRACT

The study highlights the statistical procedure employed in developing a short-term forecasting model that explores the volatility feature of Philippine inflation from 1995 up to August 2007. To build such a model, we identify first the stationary series. Second, we specify the *Autoregressive Moving Average* (ARMA) model then include the *Seasonal ARMA* (SARMA) model if seasonality is present, to represent the mean component using the past values of inflation. Next, we incorporate the *Generalized Autoregressive Conditional Heteroskedasticity* (GARCH) model to represent its volatility. Diagnostic tests and examination of forecast accuracy measures indicate that the specifications D(IR) (the first difference of month-on-month inflation) as the stationary series, AR(1) and SMA(12) for the mean, GARCH(0,1) or ARCH(1) for the variance with Student's t distribution having fixed degrees of freedom $v = 10$ for the errors, performs best in forecasting the volatility of the inflation rate for the Philippines. Lastly, out of sample forecasts for the mean and error variance of Philippine inflation from September 2007 to October 2007 are achieved using dynamic forecasting.

Key words: C5, E3
JEL Classification: inflation, forecasting, model construction and estimation,
model evaluation and selection

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Forecasting the Volatility of Philippine Inflation Using GARCH Models

Haydee L. Ramon*

January 2008

1 Introduction

Autoregressive (AR), Moving Average (MA) and ARMA models are often very useful in modeling general time series.¹ However, they all have the assumption of homoskedasticity (or equal variance) for the errors. This may not be appropriate when dealing with financial market variables such as stock price indices or inflation rate. These financial market variables typically have the following three characteristics which general time series models have failed to consider:

1. The distribution of a financial time series X_t has heavier tails than normal.
2. Values of X_t do not have much correlation, but values of X_t^2 are highly correlated.
3. The changes in X_t tend to cluster. Large (small) changes in X_t tend to be followed by large (small) changes, as documented by Mandelbrot (1963).

One of the earliest time series models allowing for heteroskedasticity or time-varying variance is the *Autoregressive Conditional Heteroskedastic* (ARCH) model introduced by Engle (1982). The ARCH models have the ability to capture all the above characteristics in financial market variables. Bollerslev (1986) extended this idea into *Generalized Autoregressive Conditional Heteroskedastic* (GARCH) models which give more parsimonious results than ARCH models, similar to the situation where ARMA models are preferred over AR models.

Since then, several variations of the GARCH models have been introduced. From [21], these include:

1. ARCH-in-Mean (ARCH-M) model - presented by Engle, Lilien, and Robins (1987), which is used in financial applications where the conditional variance is related with the mean.

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¹ARMA models are used to describe the mean of the data. AR, MA, and ARMA models are expressed as the weighted average of past observation, past errors and combination of past observation and errors respectively.

2. Asymmetric ARCH model - described by Engle and Ng (1993), which is employed to account for equities often observed with downward movements in the market which are followed by higher volatilities than upward movements of the same magnitude. Two models that allow for asymmetric shocks to volatility are TARARCH and EGARCH models.
3. Threshold ARCH (TARARCH) - introduced independently by Zakoian (1990) and Glosten, Jagannathan and Runkle (1993), which considers the leverage effect on the ARCH models including the positive and negative terms on the conditional variance.
4. Exponential GARCH (EGARCH) - proposed by Nelson (1991), which implies that the leverage effect is exponential.
5. SWARCH - presented by Hamilton and Susmel (1994), which combines the ARCH models with the Markov switching method, allowing a variety of different regime changes in the ARCH models.

The following studies refer specifically to forecasting inflation in the Philippines:

1. Mariano [22] explains a statistical procedure for forecasting monthly inflation as measured by the changes in the CPI. The procedure used a price equation which was specified in terms of its own past values, and including cost-push and demand-pull factors. The CPI series over the period January 1972 to March 1985 was fitted using the ARMA process, through which short-term inflation forecasts were made from June to December 1985. This approach anticipates the subsequent work of Mariano in specifying the BSP's Single-Equation Model (SEM) for inflation forecasting.
2. Mariano, Dakila Jr. and Claveria [27] presented a structural long-term inflation forecasting model for the Philippines. The forecasting model serves as a quantitative tool to forecast headline and core inflation rates one to two years into the future. It consists of a simultaneous system of 38 estimated equations using the annual data over 1971-1999. Moreover, it provides more detail on the determination of prices in the economy.

The first paper used only ARMA models without the GARCH specification. The second paper considered the structural type of forecasting model, which is used for long-term forecasting.

The current paper develops a short-term forecasting model that explores the volatility feature of Philippine inflation from years 1995 up to 2007. To build such a model, we specify first an ARMA model, and then consider the SARMA model, if seasonality is present, to represent the mean component using the past values of inflation.² Next, we incorporate a GARCH model to represent its volatility. After the models with significant terms had been derived, the models were evaluated using static forecasting, and were then evaluated further with respect to predictive accuracy using the Diebold-Mariano (DM) test statistic. Once the best model has been found, it can be integrated (mean and variance introduced into the model) and point forecasts can be computed using dynamic forecasting. The GARCH terms representing the volatility of the process are then computed for each of the point forecast.

²SARMA or Seasonal ARMA models are recommended for monthly or quarterly data with systematic seasonal movements.[12] It can also be viewed as an extension of the ARMA process, containing only the weighted average of seasonally-spaced past observations and errors.

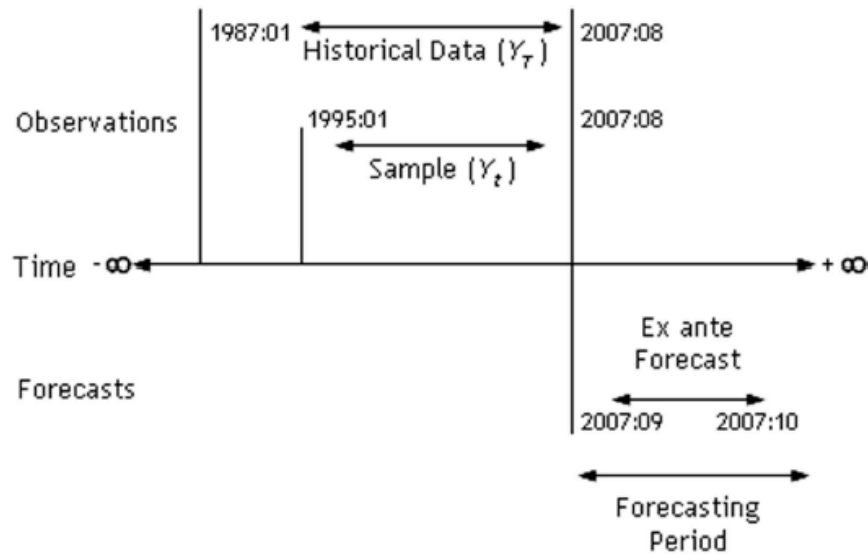


Figure 1: The Time Line Horizon in Forecasting the Volatility of Inflation Rate

2 Results and Analysis

In this section, we perform the actual forecast building using GARCH models. We do so by completing and implementing the following stages: *identification of a stationary series, estimation of GARCH models, examination of forecast accuracy measures, and dynamic forecasting.*

For clarity of discussion, we identify several conventions. We define the inflation rate (IR) of the Philippines as the annual percentage increase in the Consumer Price Index or CPI, computed by the National Statistics Office (NSO), with the year 2000 as the base period. We impose a uniform 10% level of significance on the results of the statistical tests conducted using EViews 5.1 software. For the sample period and forecasting horizon, we use the timeline presented in figure 1.

2.1 Stage 1: Identification of a Stationary Series

The line graph of the leveled IR series (figure 2) indicates the non-stationarity of the series, since volatile values are evident and these do not fluctuate around a constant mean. We thus examine the first differences of the series.³ The graph of the first differences, denoted by $D(IR)$ in figure 3, shows that the series appears to be stationary since it has no persistent trend and its values fluctuate around a constant mean of zero.⁴

³The first differences is equal to the difference of two consecutive year-on-year inflation, i.e. $IR_{Aug2007} - IR_{June2007} = D(IR)_{AugandJul2007}$.

⁴If the first differences is not mean stationary, we may consider taking the second differences which is equal to the difference of the first differenced series.

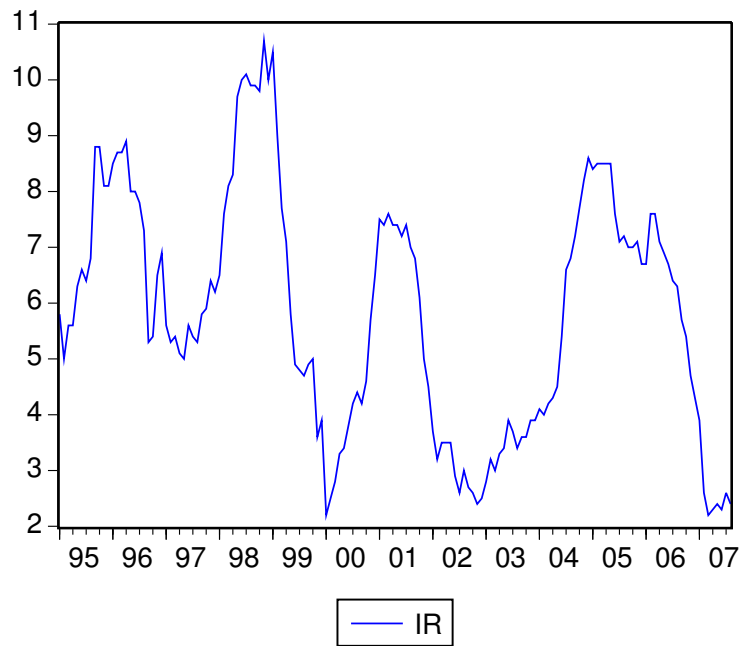


Figure 2: Line Graph of the Leveled Inflation Rate

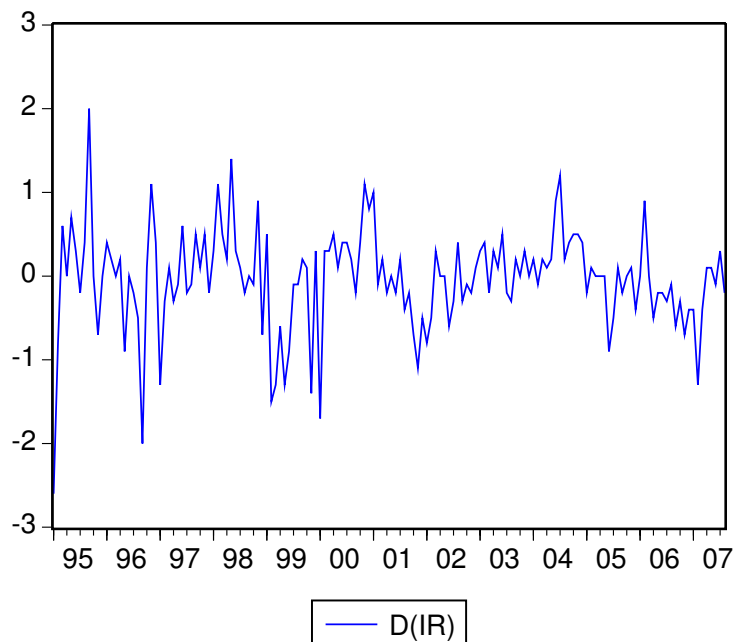


Figure 3: Line Graph for the First Differences of the Inflation Rate Series

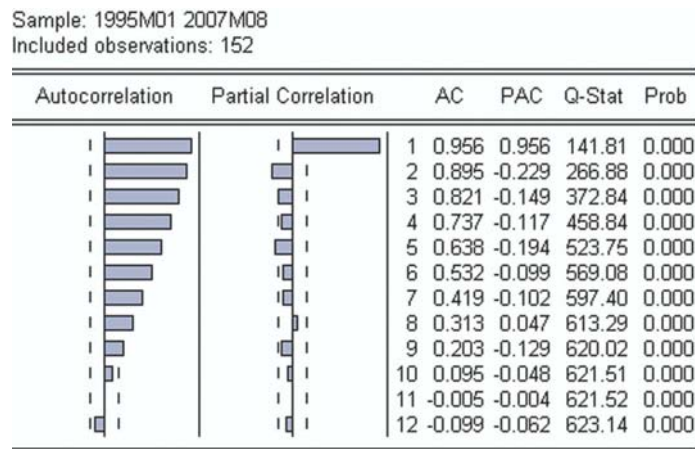


Figure 4: Correlogram of the Levelled Inflation Rate

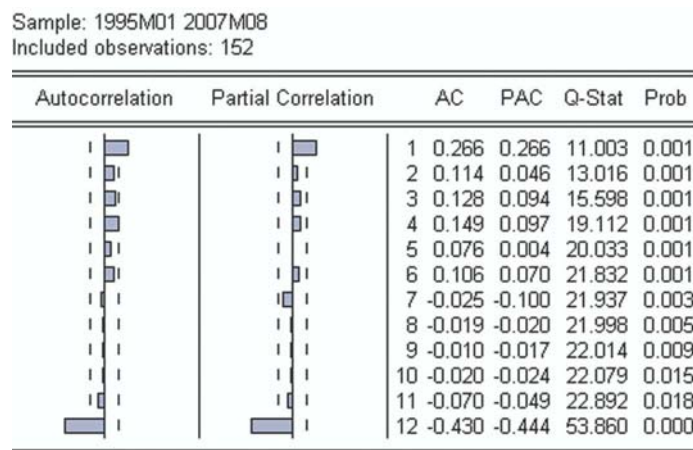


Figure 5: Correlogram of the First Differences of the Inflation Rate Series

The stationarity condition of the series can be formally verified by using the correlogram⁵ and the unit root test (URT)⁶ for the leveled and first differences of the IR series.

⁵**Correlogram** is a pictorial representation that displays the autocorrelation and partial autocorrelation functions up to a specified order of k lags. To test if autocorrelation coefficient $\rho = 0$ using the **Bartlett's Test** (which alternatively indicates stationarity if most ρ s are equal to 0) [11], we consider the following: 1. $H_0: \rho_k = 0$; 2. $H_1: \rho_k \neq 0$; 3. Test Statistic: $\frac{\hat{\rho}_k}{1/\sqrt{T}} \sim N(0, 1)$; 4. Confidence Interval: $(-z_{\alpha/2} \frac{1}{\sqrt{T}}, z_{\alpha/2} \frac{1}{\sqrt{T}})$; 5. Decision: If the computed $\hat{\rho}_k$ lies outside the confidence interval then reject the hypothesis that $\rho = 0$. This means that there is strength in the statistical relationship between ordered pairs of D(IR) separated by k periods. Otherwise, do not reject the hypothesis; where ρ_k is the autocorrelation coefficient separated by k time periods, T is the length of the series, and $z_{\alpha/2}$ is the z-value with $\alpha/2$ degrees of freedom.

⁶To test for nonstationarity $\gamma = 0$ using the **Dickey-Fuller Test** [11], we use the following: 1. Test Equation: $\Delta Y_t = \beta_t + \gamma Y_{t-1} + \epsilon_t$ where β_t is a constant; 2. $H_0: \gamma = 0$ (series is nonstationary); 3. $H_1: \gamma < 0$ (series is stationary); 4. Test Statistic: $\tau = \frac{\hat{\gamma}}{se(\hat{\gamma})}$; 5. Decision: If $|\tau|$ is less than the absolute value of the MacKinnon critical

The results on the correlogram of the leveled inflation rate series in figure 4, shows stronger evidence of non-stationarity since the autocorrelation coefficient function (ACF) of the residuals does not quickly decay to zero. On the other hand, the correlogram of D(IR) in figure 5 is consistent with mean stationarity because most of the values promptly decay to zero. However, spikes at lags 1 and 12 are evident.

Next, we test for a unit root using the augmented Dickey-Fuller (ADF) statistic. As shown in figures 6 and 7. IR is mean stationary at the 5% level of significance while D(IR) are mean stationary at the 1% level of significance.⁷ Thus, we consider D(IR) as our stationary series in the estimation of GARCH models.

Null Hypothesis: IR has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Fixed)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-3.594838	0.0335
Test critical values: 1% level	-4.019561	
5% level	-3.439658	
10% level	-3.144229	

*MacKinnon (1996) one-sided p-values.

Figure 6: Unit Root Test Output for the Leveled Inflation Rate Series

Null Hypothesis: D(IR) has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 4 (Fixed)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-4.154129	0.0066
Test critical values: 1% level	-4.019561	
5% level	-3.439658	
10% level	-3.144229	

*MacKinnon (1996) one-sided p-values.

Figure 7: Unit Root Test Output for the First Differences of the Inflation Rate Series

Dependent Variable: D(IR)
Method: Least Squares
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 15 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.159634	0.072289	2.208268	0.0288
AR(12)	-0.437014	0.096027	-4.550942	0.0000
MA(1)	0.070993	4.60E-05	1543.420	0.0000
MA(12)	-0.197929	0.118283	-1.673355	0.0964

Figure 8: Model 1 Specification for the Mean Equation under the Method of Least Squares using the Terms D(IR), AR(1), AR(12), MA(1), and MA(12)

Dependent Variable: D(IR)
Method: Least Squares
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 2 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.195667	0.064869	3.016357	0.0030
AR(12)	-0.502087	0.058224	-8.623368	0.0000

Figure 9: Model 2 Specification for the Mean Equation under the Method of Least Squares using the Terms D(IR), AR(1), MA(1) and MA(12)

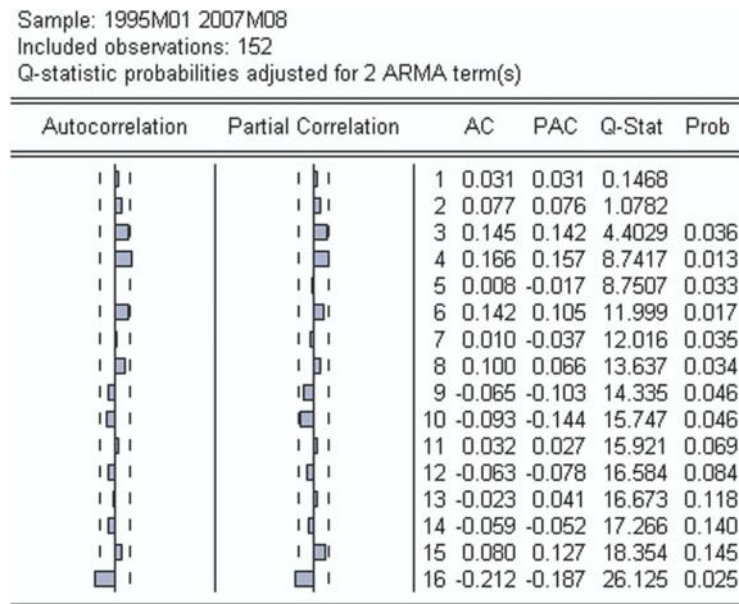


Figure 10: Correlogram Q-test of Residuals under Model 2

2.2 Stage 2: Estimation of GARCH Models

To generate parameter estimates for the GARCH model, we first construct ARMA and SARMA models to estimate the mean component by analyzing the correlogram.⁸ As can be seen in figure 5, stationary series D(IR) exhibited spikes on the terms AR(1), AR(12), MA(1), and MA(12). The ordinary least squares estimates of those terms with spikes are shown in figure 8. Non-significant terms (the ones with p-values not less than 0.10) are then removed starting to the one with the highest p-value. This generates model 2 with terms AR(1) and AR(12) for the mean equation as shown in figure 9.

value at α , do not reject the null hypothesis that the series is nonstationary, otherwise, reject the hypothesis that the series is nonstationary; where $\Delta Y_t = Y_t - Y_{t-1}$ and ϵ_t is a white noise.

⁷The ADF test statistic is lower than their critical values at the given level of significance.

⁸The spikes exhibited on the correlogram provides a rough guide to ARMA terms selection.

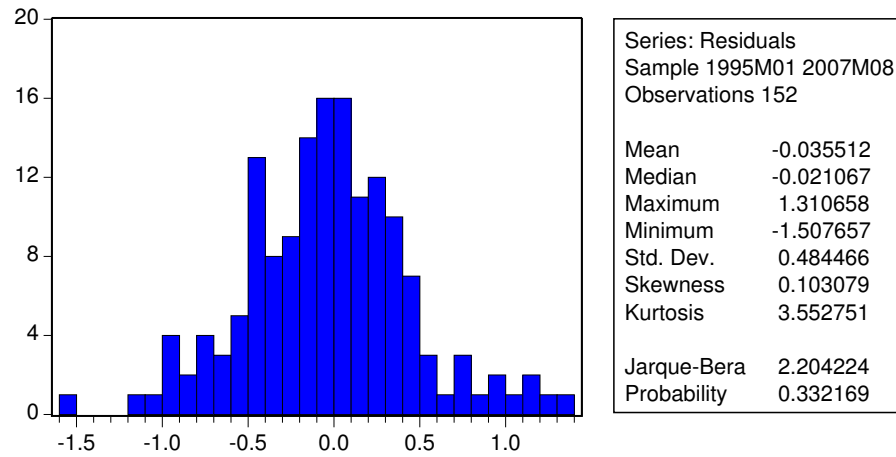


Figure 11: Histogram and Stats of Residuals under Model 2

ARCH Test:

F-statistic	0.000674	Prob. F(1,149)	0.979329
Obs*R-squared	0.000683	Prob. Chi-Square(1)	0.979156

Test Equation:

Dependent Variable: RESID^2
Method: Least Squares
Sample (adjusted): 1995M02 2007M08
Included observations: 151 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.230991	0.035975	6.420962	0.0000
RESID^2(-1)	0.002117	0.081575	0.025954	0.9793

Figure 12: ARCH Test on the Residuals under Model 2

To verify the adequacy of AR(1) and AR(12) terms of the mean equation, the (1) correlogram Q-test,⁹ (2) Jarque Bera test¹⁰ and (3) ARCH test¹¹ are used to test the residuals.

The correlogram Q-test (figure 10), indicates that there seems to be serial correlation on the residuals since the autocorrelations and partial autocorrelations at lags 4 to 12 and 16 are significant.¹² Also, if we look at the first p-value of 0.036 which is testing the null hypothesis that correlations of residuals from lags 1 to 4 are all zero, we can see that it is less than 10%. Thus, we reject H_0 and say that at least one of these correlations is not zero. On the other hand, the Jarque-Bera test as shown on the histogram and stats view of figure 11, somehow suggests normality (p-value is not significant), specifically a slight positive skewness on the residuals. Lastly, the output on the ARCH test in figure 12, signifies that we do not reject the null hypothesis that there is no ARCH up to order q in the residuals because of the insignificant squared residual term (p-value of 0.9792 is more than .10 level of significance).

Because of the result that serial correlation existed on model 2 when the correlogram Q-test is performed, we need to do a reestimating procedure. We change from model 1, MA(12) and AR(12) to SMA(12) and SAR(12) terms respectively, to take into account the seasonal component that may be present in the model. The resulting equation, which we now call as model 3, yielded an output as shown in figure 13. Then removing non-significant terms starting to the one with the highest p-value, yielded model 4 with terms AR(1) and SMA(12) for the mean, as shown in figure 14.

We now verify the validity of model 4 of the mean equation using three tests on the residuals. The correlogram Q-test of figure 15 indicates that there seems to be no serial correlation on the residuals since the autocorrelations and partial autocorrelations at all lags are nearly zero.¹³ Also the first p-value of 0.160 is greater than 0.10 which means that we do not reject H_0 and say that all correlations from lags 1 to 3 are all zero. Result of the Jarque Bera test statistic

⁹To test for the joint hypothesis of no autocorrelation up to order n using the **Ljung-Box Test** [11], we consider the following: 1. $H_0: \rho_1 = \rho_2 = \dots = \rho_n = 0$; 2. H_1 : There is a $\rho_k \neq 0$; 3. Test Statistic: $Q_{LB} = T(T+2) \sum_{k=1}^n \frac{\hat{\rho}_k^2}{T-k}$; 4. Decision: If the computed Q_{LB} exceeds the critical value from the $\chi^2(n)$ from the table at the level α , reject the null hypothesis that all autocorrelation coefficients up to order n are zero; otherwise, do not reject the hypothesis; where n is the number of lags, $\hat{\rho}_k$ is the k^{th} autocorrelation, T is the number of observations.

¹⁰To test the hypothesis on the Normality of Residuals using the **Jarque-Bera Test** [11], we consider the following: 1. H_0 : Residuals are normally distributed; 2. H_1 : Residuals are not normally distributed; 3. Test Statistic: $JB = \frac{n}{6} [s^2 + \frac{(K-3)^2}{4}] \sim \chi^2(2)$; 4. Decision: If JB is greater than the critical χ^2 value at a given level of significance α , we reject the null hypothesis of normality; where n is the number of observations, s is the skewness, and K is the kurtosis.

¹¹To test the hypothesis of no **ARCH** up to order q in the residuals [12], we consider the following: 1. H_0 : There is no ARCH up to order q ; 2. H_1 : There is ARCH up to order q ; 3. Test Regression: $\epsilon_t^2 = \beta_0 + \left(\sum_{k=1}^q \beta_k \epsilon_{t-k}^2 \right) + v_t$; 4. Test Statistic: Obs*R-squared = (No. of observations) \times (R^2 from the test regression); 5. Decision: If the p-value is less than the level of significance α , then we reject the null hypothesis that there is no ARCH up to order q in the residuals; where ϵ_t^2 is a regression of the squared residuals on a constant and lagged squared residuals up to order q , and v_t is the residual of ϵ_t^2 .

¹²p-values are less than 10% level of significance

¹³except at lag 4 with p-value equal to 0.039 which is less than 10% level of significance

Dependent Variable: D(IR)
Method: Least Squares
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 22 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.193723	0.096407	2.009435	0.0463
SAR(12)	0.105956	0.070458	1.503814	0.1348
MA(1)	0.042418	0.048850	0.868332	0.3866
SMA(12)	-0.934882	0.011464	-81.55083	0.0000

Figure 13: Model 3 Specification for the Mean Equation under the Method of Least Squares using the Terms D(IR), AR(1), SAR(12), MA(1), and SMA(12)

Dependent Variable: D(IR)
Method: Least Squares
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 10 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.228895	0.079711	2.871568	0.0047
SMA(12)	-0.932124	0.011118	-83.83751	0.0000

Figure 14: Model 4 Specification for the Mean Equation under the Method of Least Squares using the Terms D(IR), AR(1) and SMA(12)

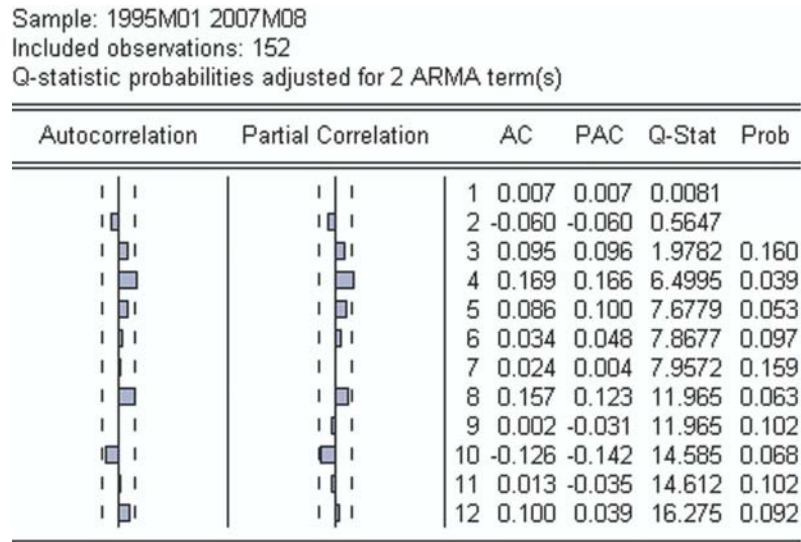


Figure 15: Correlogram Q-test of Residuals under Model 4

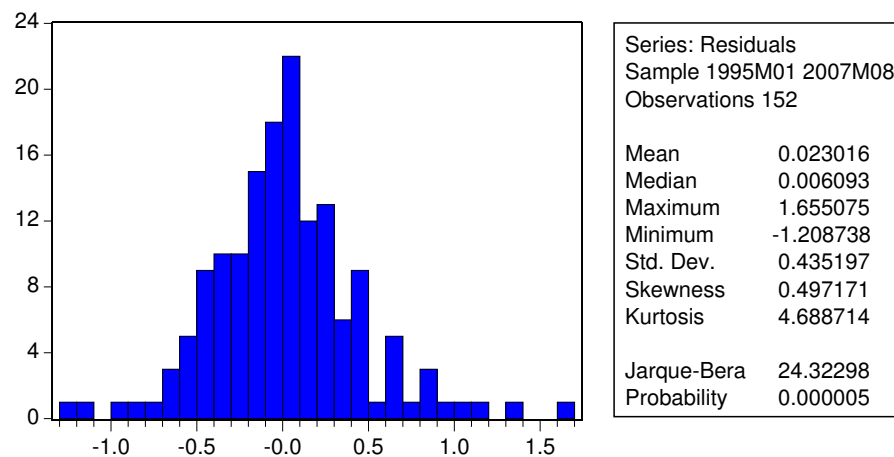


Figure 16: Histogram and Stats of Residuals under Model 4

ARCH Test:

F-statistic	1.813971	Prob. F(1,149)	0.180078
Obs*R-squared	1.816208	Prob. Chi-Square(1)	0.177765

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Sample (adjusted): 1995M02 2007M08
Included observations: 151 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.166259	0.033662	4.939070	0.0000
RESID^2(-1)	0.109516	0.081313	1.346837	0.1801

Figure 17: ARCH Test on the Residuals under Model 4

as shown on the histogram and stats view of figure 16 somehow suggests non-normality since the statistic is large, and slight positive skewness on the residuals. Lastly, the output on the ARCH test as shown in figure 17 signifies that we do not reject the null hypothesis that there is no ARCH up to order q in the residuals because of the insignificant squared residual term (p-value of 0.1778 is more than .10 level of significance).

GARCH(p, q)	Type of Error Distribution
GARCH(0, 1)	Normal Distribution
GARCH(0, 1)	Student's t with fixed degrees of freedom (d.f.) $v = 10$ (default value)
GARCH(0, 1)	Generalized Error Distribution (GED) with fixed parameter (f.p.) $r = 1.5$ (default value)

Table 1: Significant GARCH(p, q) Models and their Error Distributions

The result of no serial correlation under the correlogram Q-test, using the AR(1) and SMA(12) terms for the mean equation, indicates that we can now proceed with the estimation of the conditional variance for the errors using GARCH. We limit the order of GARCH(p, q) to 4, that is we use different orders of $p, q=0,1,2,3$ and 4 or four months relationship of volatilities, since GARCH is used for short-term forecasting. Incorporating the stationary series D(IR) and the mean equation with terms AR(1) and SMA(12), we estimate a GARCH model by finding a significant order combination under a specific error distribution.¹⁴ After testing different orders of p and q , it was found that the significant orders at the 10% level includes GARCH(0, 1)¹⁵

¹⁴p-values should all be less than .10 level of significance and coefficient of the variance equation should all be positive.

¹⁵GARCH(0, 1) is also equal to ARCH(1)

Dependent Variable: D(IR)
Method: ML - ARCH (Marquardt) - Normal distribution
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 64 iterations

GARCH = C(3) + C(4)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.346004	0.095157	3.636146	0.0003
MA(12)	-0.948621	0.006525	-145.3885	0.0000
Variance Equation				
C	0.146448	0.015576	9.402425	0.0000
RESID(-1)^2	0.247463	0.111423	2.220936	0.0264

Figure 18: GARCH(0,1) Volatility Model of the Inflation Rate assuming a Normal Distribution for the Error Terms

assuming the error distributions in table 1. The output results are shown in figures 18, 19, and 20 for verification.

2.3 Stage 3: Examination of Forecast Accuracy Measures

GARCH(<i>p, q</i>)	Error Dist'n	RMSE	MAE	MAPE
GARCH(0, 1)	Normal	0.440042	0.319337	5.975077
GARCH(0, 1)	Student's t with fixed d.f. <i>v</i> =10	0.437680	0.316931	5.941765
GARCH(0,1)	GED with f.p. <i>r</i>=1.5	0.437332	0.316818	5.947739

Table 2: Forecast Errors for the Significant GARCH(*p, q*) Models

To validate the goodness of fit of GARCH(0,1) model assuming normal, Student's t with fixed degrees of freedom $v = 10$ and GED with fixed parameter $r = 1.5$ distributions for the error terms, we perform *static* forecasting on the models to show measures of forecast accuracy¹⁶ over the estimation period. Table 2 shows the summary results of forecast accuracy. We observe that in terms of RMSE and MAE, GED with fixed parameter $r = 1.5$ formulates the model with the smallest measure of forecast error, that is, the one with the most accurate fit

¹⁶As a general rule, the smaller the sum of the absolute errors $\sum_{t=1}^n |\epsilon_t|$ or the sum of the squared errors $\sum_{t=1}^n \epsilon_t^2$, the more accurate the fit of the model. The following is a summary of statistical measures of forecast accuracy

of a model: 1. **Mean Absolute Error or MAE** = $\frac{\sum_{t=1}^n |\epsilon_t|}{n}$; 2. **Mean of the Absolute Percentage Error or MAPE**

Dependent Variable: D(IR)
Method: ML - ARCH (Marquardt) - Student's t distribution
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 37 iterations

t-distribution degree of freedom parameter fixed at 10
GARCH = C(3) + C(4)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.292095	0.087509	3.337895	0.0008
MA(12)	-0.947792	0.006753	-140.3466	0.0000
Variance Equation				
C	0.134234	0.019925	6.736845	0.0000
RESID(-1)^2	0.237624	0.131363	1.808917	0.0705

Figure 19: GARCH(0,1) Volatility Model of the Inflation Rate assuming a Student's t with fixed degrees of freedom $v = 10$ for the Error Terms

Dependent Variable: D(IR)
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Sample: 1995M01 2007M08
Included observations: 152
Convergence achieved after 39 iterations

GED parameter fixed at 1.5
GARCH = C(3) + C(4)*RESID(-1)^2

	Coefficient	Std. Error	z-Statistic	Prob.
AR(1)	0.295724	0.088338	3.347657	0.0008
MA(12)	-0.946465	0.006945	-136.2718	0.0000
Variance Equation				
C	0.141275	0.020408	6.922646	0.0000
RESID(-1)^2	0.227356	0.132383	1.717409	0.0859

Figure 20: GARCH(0,1) Volatility Model of the Inflation Rate assuming GED with fixed parameter $r = 1.5$ for the Error Terms

of the time series model. MAE indicates that the average difference between the forecast and the observed value of the model is 0.3168. However, GARCH(0,1) for the variance component with an assumption of GARCH(0,1) assuming a Student's t distribution with fixed d.f. $v = 10$ for the error terms formulates the model with the smallest measure of MAPE. This implies that on the average, the forecasts from the model are off by 5.9418% of the true value. If the standard chosen for forecasting is a MAPE of 10%, then this model is doing well.

Error Distributions	DM Test Statistic B
GED with fixed parameter $r = 1.5$ vs. Student's t with fixed d.f. $v = 10$	1.198866
Normal vs. Student's t with fixed d.f. $v = 10$	2.132320
Normal vs. GED with fixed parameter $r = 1.5$	1.295412

Table 3: DM Test Statistic B given Pairwise Error Distributions

We further evaluate predictive accuracy of the models using the Diebold-Mariano (DM) test statistic.¹⁷ The table indicates that at a 95% confidence interval, we reject the null hypothesis of no difference in the accuracy of the normal and Student's t with fixed d.f. $v = 10$ error distributions. The positive value of $B = 2.1323$, which is greater than the critical value of 1.96 in a normal table, indicates that the normal distribution has higher square error than Student's t with fixed degrees of freedom $v = 10$. On the other end, when Student's t with fixed degrees of freedom $v = 10$ vs. GED with fixed parameter $r = 1.5$ are compared together with the normal vs. GED with fixed parameter $r = 1.5$ error distributions, we do not reject the null hypothesis of no difference in the accuracy of the competing forecast models. These results direct us that the Student's t with fixed d.f. $v = 10$ error distribution is the most adequate choice among the significant error distributions. Although GED with fixed parameter $r = 1.5$ was found to have the smallest measure in its forecast errors based on RMSE and MAE, we lay much emphasis on the result of measuring predictive accuracy of the models through the DM test statistic. We therefore reiterate our choice of the Student's t with fixed d.f. $v = 10$ as the most adequate choice for the variance of the error distribution of Philippine inflation.

Incorporating the most adequate choice for the volatility model, we now present the forecast for the mean and error variance of the inflation rate, as shown in figure 21, using the in-sample

$$= \frac{\sum_{t=1}^n |\epsilon_t|}{n}; \text{ 3. Root Mean Square Error or RMSE} = \sqrt{\frac{\sum_{t=1}^n \epsilon_t^2}{n}}$$

where ϵ_t is the forecast error in time period t , Y_t is the actual value in time period t and n is the number of forecast observations in the estimation period.

¹⁷To test the hypothesis of $E(d_t) = 0$ using the **Diebold-Mariano Test** we consider the following: 1. $H_0: E(d_t) = 0$; 2. $H_1: E(d_t) \neq 0$; 3. Test Statistic: $B = \frac{\bar{d}}{\sqrt{\hat{f}}/T} \sim N(0, 1)$; 4. Decision: If $|B|$ is greater than the critical normal value

at a given level of significance α , we reject the null hypothesis of no difference in the accuracy of two competing forecasts; where $E(d_t)$ is the expected loss differential, \bar{d} is the sample mean loss differential, \hat{f} is the sample variance of the sample mean loss differential and T is the number of ex post observations.

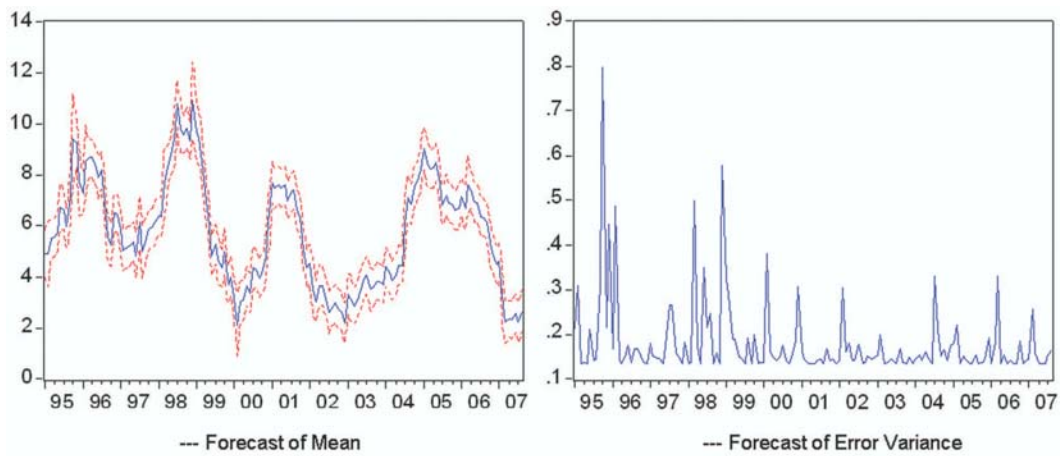


Figure 21: GARCH(0,1) Model Forecast for the Mean and Error Variance of Inflation Rate using the In-sample Observations under Static Forecasting

observations under static forecasting. The figure implies that volatile values are evident during the years between 1995 to 1996 and 1997 to 1999. This is evident in the wide confidence intervals on the GARCH model under the forecast of mean. For the year 2000 onwards, however, we observe a stable and predictable inflation rate, as shown in the low values of the forecast of error variance.

2.4 Stage 4: Dynamic Forecasting

To summarize, the stationary series D(IR), AR(1) and SMA(12) terms for the mean, and GARCH(0,1) term for the variance assuming a distribution of Student's t with fixed degrees of freedom $v = 10$, formulate the most adequate model for the volatility of the inflation rate. Thus, we can now extrapolate, that is, forecast the inflation rate beyond the historical data in two months duration. We choose dynamic forecasting in extrapolation. Figure 22 shows the forecasts for the mean and error variance of the inflation rate using the out-of-sample observations under dynamic forecasting. The forecast values for the inflation rate, denoted by IRDF, and its error variance, denoted by GARCHDF, are shown in table 4. The table also includes the lower and upper bounds at the 80% and 95% confidence intervals of the inflation rate considering the forecast horizon from September 2007 to October 2007.

Period	IRDF	GARCHDF	80% Confidence Interval		95% Confidence Interval	
			Low	High	Low	High
2007:09	2.77860	0.15679	2.26729	3.28992	2.00172	3.55549
2007:10	2.83024	0.17149	1.97954	3.68095	1.53771	4.12278

Table 4: Forecast for the Inflation Rate including the Lower and Upper Bounds from September to October 2007

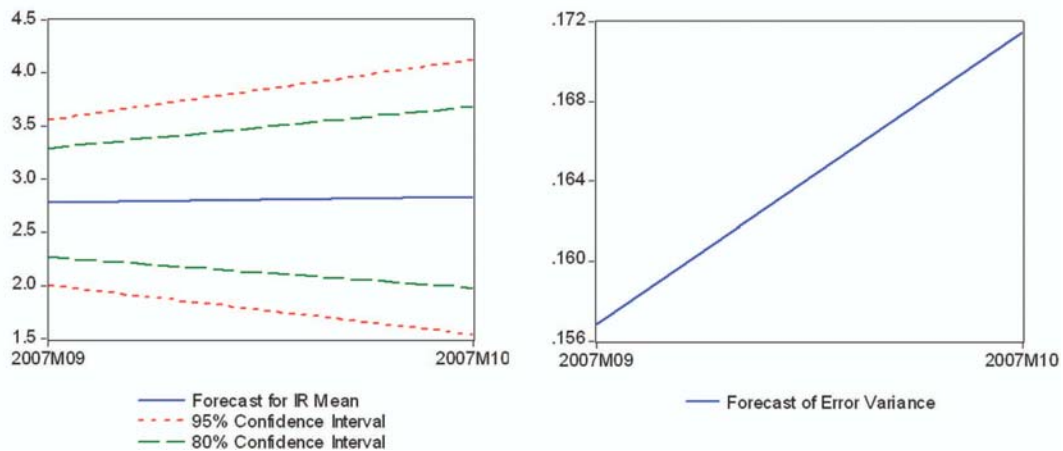


Figure 22: GARCH(0,1) Model Forecast for the Mean and Error Variance of the Inflation Rate using the Out-of-Sample Observations under Dynamic Forecasting

The ex ante forecast implies that for September and October 2007, the inflation forecast will be seen to rise slightly from 2.78% to 2.83% respectively. The projected confidence bands for September 2007 implies that the inflation rate can be as low as 2.3% to as high as 3.3% at the 80% confidence interval and as low as 2% to as high as 3.6% at the 95% confidence interval. As can be noticed, we do not forecast for far duration because the volatility increases rapidly and it becomes trivial.

The forecast for the variance of the errors or GARCHDF indicates an increasing trend, that is, as the period lapses through time, the variance of the errors increases. The percentage change in the variance from September to October 2007 signifies an 9.37% percentage increase.

3 Summary and Conclusions

This paper has proposed specifications for the mean, variance and error using ARMA, SARMA and GARCH models to predict the volatility of Philippine inflation based on the monthly data from January 1995 to August 2007.

The parameter estimation procedure started with transforming the IR data into a stationary series. After investigating the ACF and PACF plots, and conducting a formal ADF test on stationarity, it was shown that the first differences $D(IR)$ was the stationary series.

To estimate the mean component of the IR series, we derived a tentative model (known as model 1) using the least squares method and obtained the terms AR(1), AR(12), MA(1), and MA(12) through the analysis of the ACF and PACF plots. Removing non-significant terms starting to the one with the highest p-value yielded model 2 with terms AR(1) and AR(12). However, when residual tests were conducted specifically the correlogram Q-test, serial correlation existed so the conduct of a reestimating procedure was made. We changed from model

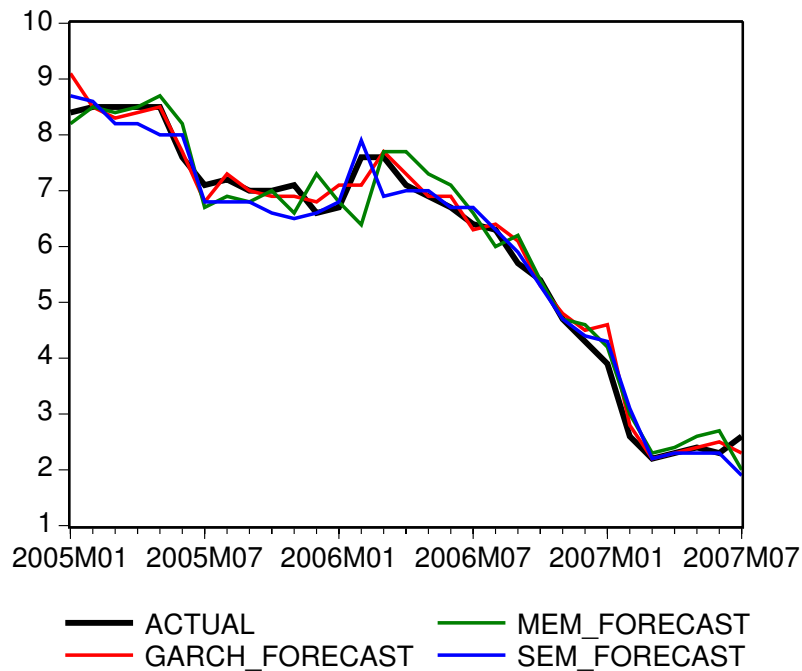


Figure 23: Historical One-Month Ahead Forecasts of SEM, MEM and GARCH Models from January 2005 to August 2007

1, MA(12) and AR(12) to seasonal terms SMA(12) and SAR(12) respectively and called it as model 3. We subsequently removed non-significant terms to achieve model 4 for the mean equation with terms AR(1) and SMA(12). We then verified the validity of model 4 using three residual tests. The correlogram Q-test indicated no serial correlation on the residuals, thus we proceeded to the next stage of examining the forecast accuracy measures.

Using the maximum likelihood method, we found that the smallest forecast error for RMSE and MAE was GARCH(0,1) assuming GED with f.p. $r = 1.5$ while the smallest forecast error for MAPE was GARCH(0,1) assuming Student's t distribution with fixed d.f. $v=10$. However, when the Diebold-Mariano test was conducted to compare predictive accuracy of the models, it was considered that the Student's t with fixed degrees of freedom $v = 10$ was the most adequate choice for the variance of the error distribution of Philippine inflation. We therefore choose GARCH(0,1) assuming a Student's t distribution with fixed degrees of freedom $v = 10$ as the final specification for the error variance.

To summarize, the stationary series D(IR), AR(1) and SMA(12) terms for the mean, and GARCH(0,1) term for the variance assuming Student's t with fixed degrees of freedom $v = 10$ for the error terms, yielded the most adequate model formulation for the volatility of Philippine inflation during the period January 1995 to August 2007.

Dependent Variable: ACTUAL
Method: Least Squares
Sample: 2005M01 2007M07
Included observations: 31

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SEM_FORECAST	0.312796	0.152017	2.057641	0.0490
MEM_FORECAST	0.036002	0.137995	0.260895	0.7961
GARCH_FORECAST	0.648061	0.192225	3.371366	0.0022

Figure 24: Forecasting Properties of SEM, MEM and GARCH Models

It is important to note that we cannot draw general conclusions for the inflation rate in a long-term horizon using ARMA, SARMA and GARCH models. Thus, GARCH models are essentially limited to short-term forecasting only, up to about a two-month horizon. This suggests that a GARCH model would have to be continually reestimated to consider the most recent observations.

While the current BSP's inflation forecasting models, such as the single equation model (SEM) and multiple equation model (MEM) assume a constant variance for the errors, the GARCH model developed in this paper allows for time-varying variance in the errors. During certain periods of high volatility, the GARCH model can thus help complement the BSP's current set of models, since the actual value of inflation is closer to the GARCH forecasts than SEM and MEM forecasts,¹⁸ as demonstrated in figure 24. In addition, when the forecasting properties of the GARCH model were compared with the two other BSP models using the test of Chong [10],¹⁹ as shown in figure 23, the results albeit preliminary showed great promise for the use of the GARCH model in the forecasting mix of the BSP for inflation. At present, a more comprehensive paper is being developed to further refine and strengthen the models. In such case, GARCH model will be incorporated in the present forecasting models of BSP.

For now, the author hopes that this paper will serve as a guide for future studies in studying the volatility of Philippine inflation including other economic variables of interest. One may wish to consider other variance models such as TARCH, EGARCH, PARCH, and component GARCH.

¹⁸SEM and MEM forecasts are actual forecasts as contained in the inflation projection memorandum submitted to the Governor monthly, except on November 2005 and February 2006 in which RVAT adjustment was not considered in the given MEM forecast data.

¹⁹The test, in the tradition of Chong and Hendry (1986), involves the regression of the actual values of the given variable on the competing forecasts subject to the constraint that the sum of the coefficients is one. An encompassing model is found if the coefficient of its forecast is the ONLY significant one in the regression equation. This implies that all necessary information in forecasting the variable is contained in the model's forecasts and the others do not contribute significantly to forecast accuracy. If no encompassing model is found, an optimal forecast combination can be formulated using the respective coefficients as weights.

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Appendix

A The GARCH(p, q) Framework

Subsection A.1 offers a description of the GARCH(p, q) model including its main component, the conditional variance h_t . Subsection A.2 presents simpler models of GARCH such as GARCH(1,1), in which, each of its variable terms are interpreted. Lastly, subsection A.3 illustrates the parameter estimation process assuming a normal distribution for the errors.

A.1 GARCH(p, q) Model

GARCH stands for *Generalized Autoregressive Conditional Heteroskedasticity*. To understand what GARCH is, consider the meaning of its acronym word by word as described in [23]:

Generalized - It is developed by Bollerslev as a generalization of Engle's original ARCH volatility modeling technique [15].

Autoregressive - It describes a feedback mechanism that incorporates past observations into the present.

Conditional - It implies a dependence on the observations of the immediate past.

Heteroskedasticity - Loosely speaking, we can think of heteroskedasticity as time-varying variance.

GARCH then, as cited in [23], is a mechanism that includes past variances in the explanation of future variances. More specifically, GARCH is a time series technique that allows users to model and forecast the conditional variance of the errors. It is used to take into account *excess kurtosis*²⁰ and *volatility clustering*²¹, two important characteristics of financial time series.

To formally define GARCH, let $\epsilon_1, \epsilon_2, \dots, \epsilon_T$ be the time series observations denoting the errors and let F_t be the set of ϵ_t up to time T , including ϵ_t for $t \leq 0$. As defined by Bollerslev [8], "the process ϵ_t is a Generalized Autoregressive Conditional Heteroskedastic model of order p and q , denoted by GARCH(p, q), if ϵ_t given an information set F_t has a mean of zero and conditional variance h_t given by

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 + \beta_1 h_{t-1} + \dots + \beta_p h_{t-p} \\ &= \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^p \beta_j h_{t-j}. \end{aligned} \quad (1)$$

Note that the conditional variance h_t is the main component of a GARCH model and is expressed as a function of three terms namely:

²⁰**excess kurtosis** is characterized as having a fat tail behavior [23]

²¹**volatility clustering** is a characteristic of a financial time series in which large changes tend to follow large changes, and small changes tend to follow small changes in the time series data [28]

$$\begin{aligned} \alpha_0 & \text{ - a constant term} \\ \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 & \text{ - ARCH term} \\ \sum_{j=1}^p \beta_j h_{t-j} & \text{ - GARCH term.} \end{aligned}$$

We define ϵ_{t-i}^2 as the past i period's squared residual from the mean equation while the h_{t-j} is the past j period's forecast variance. The order of the GARCH term and ARCH term are denoted by p and q respectively. The unknown parameters which needs to be estimated are α_0 , α_i and β_j , where $i = 1, \dots, q$ and $j = 1, \dots, p$. To guarantee that the conditional variance $h_t > 0$, it needs to satisfy the following conditions: $\alpha_0 > 0$, $\alpha_i \geq 0$, and $\beta_j \geq 0$."

Based on [12], "there are five distributions for the errors, ϵ_t commonly used when working with GARCH models in EViews: *Normal (Gaussian)*, *Student's t*, the *Generalized Error (GED)*, the *Student's t with fixed degrees of freedom v*, or the *GED with fixed parameter r*. The last two error distributions are special cases of the Student's t and GED wherein the user will have the option to enter a value for the fixed parameter". We now present the first 3 log-likelihood functions denoted as l_t for the error terms.

From [23], "if ϵ_t is normally distributed,

$$l_t = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^T \log h_t - \frac{1}{2} \sum_{t=1}^T \frac{\epsilon_t^2}{h_t} \quad (2)$$

where T is the sample size.

If ϵ_t is *Student's t*,

$$l_t = T \log \left[\frac{\Gamma(\frac{v+1}{2})}{\pi^{\frac{1}{2}} \Gamma(\frac{v}{2})} (v-2)^{-\frac{1}{2}} \right] - \frac{1}{2} \sum_{t=1}^T \log h_t - \frac{v+1}{2} \sum_{t=1}^T \log \left[1 + \frac{\epsilon_t^2}{h_t(v-2)} \right] \quad (3)$$

where T is the sample size and v is the degrees of freedom."

Based on [2], "if ϵ_t is GED,

$$l_t = T \left[\log \frac{r}{\lambda} - \left(1 + \frac{1}{r}\right) \log(2) - \log\left(\Gamma\left(\frac{1}{r}\right)\right) \right] - \frac{1}{2} \sum_{t=1}^T \left| \frac{\epsilon_t}{\lambda \sqrt{h_t}} \right|^r - \frac{1}{2} \sum_{t=1}^T \log h_t \quad (4)$$

where T is the sample size, $\Gamma(\cdot)$ is the gamma function, λ is a constant given by

$$\lambda = \left[\frac{2^{-\frac{2}{r}} \Gamma\left(\frac{1}{r}\right)}{\Gamma\left(\frac{3}{r}\right)} \right]^{\frac{1}{2}} \quad (5)$$

and r is a positive parameter governing the thickness of the tails of the distribution. When $r = 2$, and $\lambda = 1$ then GED is the standard normal distribution".

A.2 GARCH(1,1)

According to [12], “the most widely used GARCH(p, q) model is the GARCH(1,1). This model has the main features which are present in more elaborate models and often fits almost well. [12] The specification of the model is given by

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}. \quad (6)$$

The conditional variance h_t is the one-period ahead forecast variance based on past information. It is a function of three terms:

- α_0 - a constant term
- ϵ_{t-1}^2 - the ARCH term, this is the news about volatility from the previous period, measured as the lag of the squared residual from the mean equation
- h_{t-1} - the GARCH term, it is the last period's forecast variance.

The (1, 1) in GARCH(1, 1) refers to the presence of a first-order GARCH term (the first term in parentheses) and a first-order ARCH term (the second term in parentheses). We can interpret the period's variance as the weighted average of a long term average (the constant), the forecasted variance from last period (the GARCH term), and information about the volatility observed in the previous period (the ARCH term).”

A.3 ARCH(q)

From [15], “the ARCH model is a special case of a GARCH specification in which, there are no GARCH terms in the conditional variance equation from (1). Thus ARCH(q)=GARCH(0, q). The process ϵ_t is an *Autoregressive Conditional Heteroskedastic* process of order q or ARCH(q), if h_t is given by:

$$\begin{aligned} h_t &= \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \dots + \alpha_q \epsilon_{t-q}^2 \\ &= \alpha_0 + \sum_{i=1}^q \alpha_i \epsilon_{t-i}^2 \end{aligned} \quad (7)$$

where $q > 0$ and $\alpha_0 > 0$, and $\alpha_i \geq 0$ for $i = 1, \dots, q$. Again, the conditions $\alpha_0 > 0$ and $\alpha_i \geq 0$ are needed to guarantee that the conditional variance $h_t > 0$.

To carry out the process of parameter estimation, consider the simplest model which is the GARCH(0,1) model, where h_t is given by:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2. \quad (8)$$

Based on [29], “the parameters α_0 and α_1 can be approximated by *maximum likelihood estimation* or MLE²². The *likelihood* L of a sample of n observations x_1, x_2, \dots, x_n , is the joint probability function $p(x_1, x_2, \dots, x_n)$ when x_1, x_2, \dots, x_n are discrete random variables. If x_1, x_2, \dots, x_n are continuous random variables, then the *likelihood* L of a sample of n observations, x_1, x_2, \dots, x_n , is the joint density function $f(x_1, x_2, \dots, x_n)$. Let L be the *likelihood* of a sample, where L is a function of the parameters $\theta_1, \theta_2, \dots, \theta_k$. Then the *maximum likelihood estimators* of $\theta_1, \theta_2, \dots, \theta_k$ are the values of $\theta_1, \theta_2, \dots, \theta_k$ that maximize L . Let θ be an element of Ω . If Ω is an open interval, and if $L(\theta)$ is differentiable and assumes a maximum on W , then MLE will be a solution of the equation $\frac{\partial L(\theta)}{\partial \theta} = 0$.”

To illustrate MLE as shown in [21], “we let $\epsilon_1, \dots, \epsilon_T$ be the errors and F_t be the set of ϵ_t up to time t which is assumed to be *normally distributed*. Then the joint density of the observations $\epsilon_1, \dots, \epsilon_T$ is

$$f(\epsilon_1, \dots, \epsilon_T) = \prod_{j=2}^T f(\epsilon_j | \epsilon_1, \dots, \epsilon_{j-1}) \cdot f(\epsilon_1). \quad (9)$$

For $k = 2, \dots, T$, the conditional density is

$$f(\epsilon_k | \epsilon_1, \dots, \epsilon_{k-1}) = \frac{1}{\sqrt{2\pi(\alpha_0 + \alpha_1 \epsilon_{k-1}^2)}} \exp \left\{ -\frac{\epsilon_k^2}{2(\alpha_0 + \alpha_1 \epsilon_{k-1}^2)} \right\}. \quad (10)$$

The marginal density of ϵ_1 is dropped for simplicity, and the resulting likelihood function becomes,

$$L_t(\alpha_0, \alpha_1) = \prod_{j=2}^T \frac{1}{\sqrt{2\pi(\alpha_0 + \alpha_1 \epsilon_{j-1}^2)}} \exp \left\{ -\frac{\epsilon_j^2}{2(\alpha_0 + \alpha_1 \epsilon_{j-1}^2)} \right\}. \quad (11)$$

The log likelihood function, neglecting the constant term is

$$l_t(\alpha_0, \alpha_1) = -\frac{1}{2} \sum_{j=2}^T \log(\alpha_0 + \alpha_1 \epsilon_{j-1}^2) + \frac{\epsilon_j^2}{\alpha_0 + \alpha_1 \epsilon_{j-1}^2}. \quad (12)$$

We can find the estimates $\hat{\alpha}_0$ and $\hat{\alpha}_1$ by solving the derivatives of the log likelihood function $\frac{\partial l}{\partial \alpha_0} = 0$ and $\frac{\partial l}{\partial \alpha_1} = 0$ respectively.”

²²The **method of maximum likelihood** is a general method of estimating parameters of a population by values that maximize the *likelihood* (L) of a sample.