# Should Inequality Factor into Central Banks' Decisions?*

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**Abstract**

Inequality is increasingly a policy concern. It is well known that fiscal and structural policies can mitigate inequality. However, less is known about the potential role of monetary policy. This paper investigates how inequality matters for the conduct of monetary policy within a tractable Two-Agent New Keynesian model. We find some support for making consumption inequality an explicit target for monetary policy, particularly if central banks follow standard Taylor rules. Given the importance of labor income at the lower end of the income distribution, we also consider augmented Taylor rules targeting the labor share. We find that such a rule is preferable to targeting consumption inequality directly. However, under optimal monetary policy the gains from targeting inequality are smaller.

**JEL Codes:** E21, E32, E52.

**Keywords:** Inequality, Optimal monetary policy, Taylor rules.

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# 1 Introduction

Should central banks care about inequality when setting monetary policy? Up until recently this question was a non-starter among policy makers and academics. First, inequality is typically outside central banks' mandates. Second, an early literature showed that inequality of wealth or income did not affect the aggregate transmission of shocks.[1] More recently, the debate seems to be shifting. Over the last few years major central bank officials have increasingly discussed distributional issues.[2] At the same time, advances in economic theory have shed light on new channels through which inequality may influence the transmission of monetary policy.

This paper investigates how inequality affects *desirable* monetary policy in a stylized economy subject to technology shocks. Our model builds on Bilbiie, 2008 and Debortoli and Gali, 2018. In this setting, a rich agent owns all the capital and her income is thus composed of after-tax dividends and wages. In contrast, a poor agent receives only wages and potentially a transfer from the government financed by the tax on dividends. The fact that the rich agent has an additional source of income (dividends) leads to inequality of income and consumption in the steady state, unless all dividends are taxed away. Positive productivity shocks lead to higher profits and hence higher dividends, thereby exacerbating initial income and consumption inequality. This is further reinforced by an assumed tech-bias in wage income: when productivity rises, the rich agent's share of total wage income goes up, while the poor agent's declines. Finally, we assume that both wages and prices are subject to nominal rigidities. These mechanisms are consistent with important moments of U.S. micro- and macro-economic data and match the empirical effects of technology shocks on consumption inequality (De Giorgi and Gambetti, 2017).[3] The model assumes a simplistic wealth distribution in steady-state and no aggregate savings in equilibrium. It also abstracts from heterogeneity in the extensive margin of labor which is known to be another important driver of inequality over the cycle, although the assumption of tech-biased wage income could be thought of as capturing this channel in reduced form.

We study the implications of inequality for optimal monetary policy (Rotemberg and Woodford, 1997). Specifically, we focus on how the initial level of consumption inequality matters for the optimal monetary policy *response* to a technology shock. We focus on consumption inequality as this is the welfare-relevant concept that arises from the central bank's Ramsey maximization problem in our setting. Optimal monetary policy, though, requires a deeper knowledge of the economy than even the savviest central banker can hope to have. Hence, we also study implications for the use of augmented Taylor rules (Taylor, 1993), which is a simpler way of conducting monetary policy.

Under optimal policy, the utilitarian central bank solves a Ramsey social welfare maximization problem by adjusting the expected path of interest rates under full information and caring equally about all individuals. We find that such a central bank places some weight on observed consumption inequality and labor share, as doing so improves aggregate welfare. However, the optimal weights on consumption inequality and labor share are small compared to those on output and inflation gaps. This means that

[1] Krusell and Smith, 1998 is a prominent example. They show how the dynamics of their heterogeneous agents model are practically identical to those of a representative agent.

[2] As examples, see (i) a 2020 speech by Chair Jerome H. Powell, that links the adjustment to the employment mandate of the Federal Reserve with an appreciation for benefits to low- and moderate-income communities, (ii) an older speech by Chair Janet L. Yellen in 2014, and (iii) a speech by ECB President Mario Draghi in 2016.

[3] A separate but related literature studies the effects of monetary policy *shocks* on wealth and income inequality, and the labor share, see e.g. Dolado et al., 2021, Coibion et al., 2017, and Cantore et al., 2021.

a central bank that chooses to only maximize a welfare function that includes inflation and output gaps fares only slightly worse than the central bank that maximizes the full Ramsey problem. Note that such a central bank is fully mindful of the consequences that inequality entails for the aggregate dynamics in the economy but does not target it explicitly. Interestingly, we find that higher levels of inequality require that the central bank places a larger weight on the labor share and a lower weight on inflation. This is because in an economy with high inequality, stabilizing inequality coincides with stabilizing the labor share, since the poor depend more on wages the greater the inequality.

Under standard Taylor rules, the central bank chooses the interest rate based on the output and inflation gaps. We find that targeting the labor share through an "augmented" Taylor rule raises welfare. Under such a rule, interest rates should be set lower than otherwise following a positive technology shock. A policy of lower interest rates leads to higher wages, which benefits poor agents. Beyond lowering inequality, such a policy is also beneficial more generally because it improves inflation and growth outcomes by avoiding excessive monetary tightening in response to a positive productivity shock. These insights are derived from an economy only facing productivity shocks (TFP), but carry through when evaluating alternative Taylor rules for an economy also subject to cost-push and demand shocks. In some respects, the Federal Reserve's revised framework that places an emphasis on achieving maximum employment can be thought of as related to the idea of targeting the labor share, although in the Fed's formulation this consideration is asymmetric, while here it would be symmetric.

Naturally, our results rely on the underlying assumptions. Importantly, the role of inequality for monetary policy depends crucially on the interaction between steady-state inequality and a biased response in wages and profits to technology shocks. In our model, the biased wage response is generated by a technological bias in wages, while the biased profit distribution is created by limited fiscal distribution. The technological bias in wages is well motivated empirically in De Giorgi and Gambetti, 2017.

**Literature review.** Our paper relates to three literature strands that study: (i) Two-Agent models in a New Keynesian setup; (ii) optimal monetary policy; and (iii) the interplay of inequality and monetary policy.

First, we use a Two-Agent New Keynesian (TANK) model. An early example of a two-agent model is Campbell and Mankiw, 1989, where one agent is forward looking and consumes according to her permanent income, while the other agent follows a "rule of thumb" of consuming only her current income. Gali et al., 2007 introduce these two types of consumers in a New Keynesian framework with price rigidities. Bilbiie, 2008 builds a TANK model where one type of agent has limited participation in asset markets and where standard aggregate dynamics are affected non-linearly by the share of constrained agents. Debortoli and Gali, 2018 adds to the model of Bilbiie, 2008 the distinction between liquid and illiquid components of firms' profits and introduces an explicit transfer rule. Bilbiie, 2020 shows how the elasticity of constrained agents to aggregate income is a key parameter in business cycle amplification. Other recent TANK models include Broer et al., 2020 and Walsh, 2017, who introduce heterogeneity by assuming one agent (the capitalist) holds all claims on profits but does not supply labor, while the other agent (the worker) has no claims on profits and supplies all labor.

Second, we analyze optimal monetary policy in response to productivity shocks within an economy with a distorted steady state. The study of optimal monetary policy goes back to Rotemberg and Woodford, 1997 who propose a method for deriving "optimal" monetary policy, or one that maximizes the utility of the representative household. Erceg et al., 2000 formulate optimal monetary policy under monopolistic competition and staggered nominal contracts. Clarida et al., 1999 derive optimal policy both with and without commitment within the standard New Keynesian model. Woodford, 2002 derives a social loss function from the welfare of underlying agents to study optimal monetary policy. Important for our paper, Benigno and Woodford, 2005 show how to compute a valid quadratic approximation of the social welfare function even when the steady state is distorted, in their case by monopolistic rents. We use their method applied to our distortion — steady state inequality. [4]

Third, we combine the two aforementioned strands by analyzing how optimal monetary policy depends on the degree of income inequality in a TANK model. A related literature analyzes how agent heterogeneity affects monetary policy in TANK models. Curdia and Woodford, 2010 study how monetary policy depends on the heterogeneity of preferences for consumption smoothing and dis-utility from working. Nisticò, 2016 and Bilbiie and Ragot, 2021 study how monetary policy depends on heterogeneous asset market participation.

Our paper also relates to papers on monetary policy within the class of Heterogenous Agent New Keynesian (HANK) models.[5] In this environment, Ma and Park, 2021 study the welfare implications of augmenting a standard Taylor rule that reacts to fluctuations in the income Gini index. Nuño and Thomas, 2016 and Le Grand et al., 2021 study optimal monetary policy under the presence of heterogeneity caused by uninsurable idiosyncratic risk. Kaplan et al., 2018 show that consumption responses to monetary policy actions are chiefly driven by the indirect income effect, rather than by the direct effect through intertemporal substitution. They also show that this is not the case under a representative agent but also holds under a TANK model. Gornemann et al., 2016 study how preferences for different monetary policy rules vary across agents in a HANK where the idiosyncratic unemployment risk depends on systematic monetary policy. There, the central bank operates a Taylor rule targeting unemployment and inflation. The median-wealth household favors a stronger central bank response to unemployment compared to the wealthiest households because more unemployment stabilization provides consumption insurance. Bhandari et al., 2018 study optimal monetary policy in a HANK where agents differ also in the ability to trade assets. In this setting, the central bank has an incentive to distribute the effect of the shock more evenly across agents and this incentive outweighs usual price stability considerations. Acharya et al., 2020 find similar results in a HANK economy with constant absolute risk aversion (CARA) preferences and normally distributed shocks allowing for analytical solutions. Repeatedly, Dávila and Schaab, 2022 builds a HANK economy with heterogeneity stemming from idiosyncratic income shocks. In their model, the monetary policy maker has an ex ante incentive to use inflation to redistribute consumption towards the indebted households. Moreover, the divine coincidence breaks down (even in absence of cost-push shocks) as there is a trade-off between stabilizing inflation and output, on the one hand, and redistribution on the other. Bilbiie, 2018 analyzes optimal monetary policy in a model without steady-state inequality and finds that the weight on output stabilization becomes more important as the share of constrained agents increases.

[4] Another related paper by Chang, 2022 uses a simple political economy model augmented with wealth heterogeneity to analyze how such inequality matters for the conduct of monetary policy. He finds that the usual time inconsistency of monetary policy can be offset by having a central bank that is less egalitarian than a social planner.

[5] HANK models differ from TANK models in that they allow the share of agents on their Euler equation to be determined endogenously, which introduces a role for precautionary savings. Bilbiie and Straub, 2013 provide a middle ground between the two settings, by assuming that the shares of agents on their Euler equation is constant, but its composition varies over time thereby keeping the precautionary savings component. Bilbiie, 2020 also develops an analytical HANK model as an extension of a simpler TANK model.

Closest related to our work is Bilbiie, 2008 and Debortoli and Gali, 2018. These TANK models with price rigidities have two types of agents: Ricardians and Keynesians. Ricardian consumers have full access to bond and stock markets, while Keynesian consumers are "Hand-to-Mouth" consuming their current labor income at all times. In this setting, the central bank faces a non-trivial trade-off: it is not possible to simultaneously stabilize inflation, the output gap, and consumption inequality.

**Contribution.** This paper is the first to our knowledge to study how income inequality in the steady state affects optimal and rules-based monetary policy. Our simple setting builds on Bilbiie, 2008 and Debortoli and Gali, 2018. These papers consider transitory inequality generated over the business cycle, and we extend the analysis to also consider the presence of steady state inequality and its implications for optimal and rules-based monetary policy. Our paper further departs from these in two important dimensions: (i) we consider a reduced form of technological bias in wages to match the responses of consumption of different agents to productivity shocks in the U.S.;[6] and (ii) we assume that both wages and prices are rigid.[7] These departures aim at, on the one hand, realistically capturing features of U.S. micro-data and, on the other hand, allowing the study of desirable monetary policy in the presence of inequality in a simple, tractable setting, in which steady state inequality is controlled by a single parameter that when set to zero collapses the economy back to the standard representative agent New Keynesian model.

**Organization.** The remainder of the paper is organized as follows. In Section 2, we set up our model, which we then use in Section 3 to study whether the central bank should place a weight on inequality when conducting optimal monetary policy. In Section 4 we study the welfare implications of introducing inequality targeting in otherwise standard Taylor rules. Section 5 concludes.

# 2 A Stylized Model with Steady-state and Transitional Inequality

This Section lays out the model we use to study the interactions between inequality and monetary policy in the presence of technology shocks. This model builds on Bilbiie, 2008 and Debortoli and Gali, 2018, with the main differences being that we (i) allow for consumption inequality in the steady state; (ii) introduce a reduced form skill-bias in wages in response to technology shocks; and (iii) allow for wage rigidities. The modeling of wealth inequality is limited to an uneven distribution of profits, while there are no aggregate savings in equilibrium.

The economy is populated by a continuum of infinitely lived households, indexed by $i \in [0, 1]$. Households have identical preferences, and choose consumption and savings to maximize their lifetime utility. A fraction $\lambda$ of households — so called Keynesians — does not have access to financial markets, and hence their consumption is fully determined by their income, i.e. they are "hand-to-mouth". The remaining $1-\lambda$

[6] De Giorgi and Gambetti, 2017 find an uneven impact of TFP shocks on consumption. In particular, the right tail of the consumption distribution, comprised mostly of highly educated individuals, is much more responsive to technology shocks than other parts of the distribution.

[7] Auclert et al., 2021 argue that including sticky wages in New Keynesian models is crucial to match key empirical estimates. Our results for rules-based monetary policy in the presence of wage rigidities are related to Erceg et al., 2000 and Mankiw and Reis, 2003, who suggest that wage inflation should be considered in the central bank reaction function if wage rigidities are present. However, we show that the gains from targeting the labor share, which implicitly includes the real wage, go beyond those due only to wage rigidity when inequality is present.

fraction — Ricardians — have access to financial markets where they can trade bonds[8] and stocks. Ricardians hold all equity in the economy — an extreme form of wealth inequality which translates into income inequality as all profits are distributed as dividends. Although extreme, this proxies for the fact that equity ownership is immensely concentrated in the United States (Kuhn, Ríos-Rull, et al., 2016). Households supply labor to intermediate goods producers and receive wage income in exchange. In addition, productivity shocks are assumed biased towards the wages of Ricardian agents, a reduced form "cyclical" skill-bias technological change.

The supply side features a New Keynesian structure with monopolistic competition among intermediate goods producers and a single representative firm that combines differentiated intermediate goods into a final consumption good. Nominal rigidities are introduced in both the goods and labor markets. Households are subject to taxation in the form of i) lump sum transfers imposed to finance subsidies to intermediate goods producers to undo monopolistic distortions, and ii) redistribution policies based on dividend taxes.

## 2.1 A Ricardian Agent, $r$

A Ricardian agent, $r$, obtains utility from consumption $C_{rt}$, and disutility from labor $N_t$. She takes income $Y_{rt}$ as given and chooses consumption $C_{rt}$ and nominal bond holdings $B_{rt}$, with real value $b_{rt} = \frac{B_{rt}}{P_t}$.[9] Her objective is to maximize her lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln C_{rt} - \chi \frac{N_t^{1+\phi}}{1+\phi}\right]$, subject to the budget constraint

$$C_{rt} + b_{rt} = b_{rt-1} \frac{R_{t-1}}{\Pi_t} + Y_{rt} \tag{1}$$

where $\beta$ is a time preference discount factor, $R_t$ is the gross nominal rate on riskless bonds, $\Pi_t$ is gross inflation. Taking first order conditions yields the consumption Euler equation:[10]

$$C_{rt}^{-1} = \beta R_t E_t \left( C_{rt+1}^{-1} \frac{1}{\Pi_{t+1}} \right). \tag{2}$$

The income of a Ricardian agent is composed of labor income, net-of-tax dividend payments, taxes, and transfers:

$$Y_{rt} = \frac{1 - \lambda \left( \frac{A_t}{\bar{A}} \right)^{-\gamma}}{1 - \lambda} w_t N_t + \frac{1 - \delta}{1 - \lambda} d_t - T_p Y_t + t_{rt}. \tag{3}$$

The labor income component is proportional to aggregate labor income $w_t N_t$.[11] This proportionality allows for the possibility that TFP shocks, $A_t$, affect the labor income of Ricardians and Keynesians differently, shifting resources between the two types of agents depending on the value of $\gamma$. If $\gamma > 0$, Ricar-

[8] Bonds are in zero net supply since Ricardians are all identical, Keynesians cannot borrow and the government runs a balanced budget.

[9] We assume that incomes are taken as given to avoid that type-dependent labor supply would itself have implications for inequality.

[10] In the background there is an implicit borrowing constraint that never binds.

[11] The term $\frac{1-\lambda\left(\frac{A_t}{\bar{A}}\right)^{-\gamma}}{1-\lambda}$ can be obtained by imposing that all labor income must be distributed, given that we assume $\left(\frac{A_t}{\bar{A}}\right)^{-\gamma}$ is the share of total wage income that accrues to Keynesian agents.

dians see a larger share of labor income following a positive TFP shock. Dividends are distributed equally among Ricardian agents and taxed at a rate $\delta$.[12] Taxes $T_p Y_t$ (which are raised from households and redistributed as sales subsidies to intermediate firms) eliminate the distortions from monopolistic competition. Transfers $t_{rt}$ are chosen by the fiscal authority for redistribution purposes and are financed by the taxes on dividends.

## 2.2 A Keynesian Agent, $k$

A Keynesian agent, $k$, receives income $Y_{kt}$ and chooses consumption $C_{kt}$ to maximize their lifetime utility $E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln C_{kt} - \chi \frac{N_t^{1+\phi}}{1+\phi} \right]$, subject to the budget constraint

$$C_{kt} = Y_{kt}. \tag{4}$$

Note that a Keynesian agent does not have access to financial markets, and is therefore hand-to-mouth.[13] The Keynesian agent's income is composed of labor income and transfers, net-of taxes:

$$Y_{kt} = \left( \frac{A_t}{\overline{A}} \right)^{-\gamma} w_t N_t - T_p Y_t + t_{kt} \tag{5}$$

Note that there is no dividend income, unlike the case of Ricardians, and that if $\gamma > 0$, the share of labor income that Keynesians receive decreases following a positive TFP shock, a reduced form of skill-biased wages.

## 2.3 Labor Supply and Wage Rigidities

All workers are assumed to supply the same amount of labor, $N_t$, per the following aggregate rule,[14]

$$w_t^* = \chi N_t^{\phi} Y_t, \tag{6}$$

where $w_t^*$ is the real wage norm, or the wage that would materialize absent wage rigidities, $\phi$ is the inverse Frisch elasticity, and $\chi$ measures the relative weight for the disutility of labor. We use this labor market structure to abstract from labor market distortions introduced indirectly by income inequality, avoiding, for example, that type-dependent labor supply would itself have implications for inequality. This is equivalent to the standard assumption of having a labor union that decides the amount of hours to be supplied by workers.[15]

Following Hall, 2005, wages are subject to a reduce form of nominal rigidities, where the actual nomi-

[12] Because the dividend tax and redistribution policy are isomorphic in our model, we later set $\delta = 1$ to simplify the algebra.

[13] See Bilbiie, 2008 among others. The fact that Keynesians are hand to mouth can be micro founded by different discount factors, with Keynesians being less patient than Ricardian, together with a borrowing constraint. Some examples that make such assumptions are Eggertsson and Krugman, 2012, and Benigno et al., 2020.

[14] This rule corresponds to the intratemporal optimality condition that would arise in a representative agent model, where $C_{it} = Y_t$.

[15] In our case, we do not consider a continuum of unions that differ due to the Calvo shocks, unlike in Erceg et al., 2000.

nal wage rate is a geometric combination of the past wage rate and the wage norm,

$$
\begin{aligned}
W_t &= W_{t-1}{}^{\psi_w} W_t^{*\,1-\psi_w}, \\
w_t &= \left(\frac{w_{t-1}}{\Pi_t}\right)^{\psi_w} w_t^{*\,1-\psi_w},
\end{aligned} \tag{7}
$$

where $\psi_w$ is the degree of nominal wage rigidity. This structure of nominal rigidities nests the flexible wage case (when $\psi_w = 0$).[16]

## 2.4 Markets for Goods

**Final Good Producer** The final goods market is assumed to be perfectly competitive, with a representative firm combining intermediate goods into a single final good. The production function is CES, as is the price index and the implied demand functions for intermediate inputs:

$$
Y_t = \left[\int_0^1 y_{jt}^{\frac{\theta_p - 1}{\theta_p}} dj\right]^{\frac{\theta_p}{\theta_p - 1}}, \qquad P_t = \left[\int_0^1 p_{jt}^{1-\theta_p} dj\right]^{\frac{1}{1-\theta_p}}, \qquad y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta_p} Y_t.
$$

**Intermediate Goods Producers** Each intermediate goods producer $j$ maximizes profits subject to a quadratic price adjustment cost taking as given the demand function $y_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta_p} Y_t$, as well as the real wage $w_t$, and the production function $y_{jt} = A_t L_{jt}^{1-\alpha}$ . Intermediate goods are subject to sales subsidies (at a rate $T_p$). These subsidies are imposed to undo the distortions created by monopolistic competition in steady state.

Intermediate goods producer $j$ faces the following problem at time $t$:

$$
\begin{aligned}
\max_{p_{jt}} \; & (1+T_p) \frac{p_{jt}^{1-\theta_p}}{P_t^{1-\theta_p}} Y_t - \frac{w_t}{A_t^{\frac{1}{1-\alpha}}} \left(\frac{p_{jt}}{P_t}\right)^{\frac{-\theta_p}{1-\alpha}} Y_t^{\frac{1}{1-\alpha}} - \frac{\psi_p}{2} Y_t \left[\frac{p_{jt}}{p_{jt-1}} - 1\right]^2 + \\
& + \beta E_t \left(\frac{C_{rt+1}}{C_{rt}}\right)^{-1} \left\{ (1+T_p) \frac{p_{jt+1}^{1-\theta_p}}{P_{t+1}^{1-\theta_p}} Y_{t+1} - \frac{w_{t+1}}{A_{t+1}^{\frac{1}{1-\alpha}}} \left(\frac{p_{jt+1}}{P_{t+1}}\right)^{\frac{-\theta_p}{1-\alpha}} Y_{t+1}^{\frac{1}{1-\alpha}} - \frac{\psi_p}{2} Y_{t+1} \left[\frac{p_{jt+1}}{p_{jt}} - 1\right]^2 \right\}
\end{aligned}
$$

Assuming symmetry across firms ($p_{jt} = p_{j't} = P_t$), the first order condition boils down to the New Keynesian Phillips curve,

$$
\Pi_t \left[\Pi_t - 1\right] = \beta E_t \left(\frac{C_{rt+1}}{C_{rt}}\right)^{-1} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} \left[\Pi_{t+1} - 1\right] + \frac{\theta_p}{\psi_p} \left[\frac{1}{1-\alpha} \frac{w_t}{A_t^{\frac{1}{1-\alpha}}} Y_t^{\frac{\alpha}{1-\alpha}} - (1+T_p) \frac{\theta_p - 1}{\theta_p}\right]. \tag{8}
$$

Symmetry also leads to the following aggregate production function and an aggregate equation for real dividends.

$$
Y_t = A_t N_t^{1-\alpha} \tag{9}
$$

$$
d_t = (1+T_p) Y_t - w_t N_t - \frac{\psi_p}{2} Y_t \left(\Pi_t - 1\right)^2 \tag{10}
$$

[16]Such a wage rigidity is also used by Challe et al., 2017.

### 2.5 Market Clearing

Goods and bond markets clear:[17]

$$Y_t = \lambda C_{kt} + (1-\lambda) C_{rt} + \frac{\psi_p}{2} Y_t (\Pi_t - 1)^2 \tag{11}$$

$$\int b_{it} di = 0. \tag{12}$$

### 2.6 Fiscal Policy

Fiscal policy has two independent parts. First, a sale subsidy, $T_p$, is financed by lump sum taxes on all households and is distributed to intermediate producers.[18] Second, profits are distributed according to the following rules:

$$t_{kt} = (1-\tau) d_t \tag{13}$$

$$t_{rt} = \frac{d_t - \lambda t_{kt}}{1-\lambda}. \tag{14}$$

$1-\tau$ represents the extent to which taxed profits $\delta d_t$ are distributed to Keynesians. Thus $\tau$ controls income inequality after taxes and transfers in steady-state, as well as over the business cycle.[19] A higher value of $\tau$ thus increases inequality, as less profits are redistributed from Ricardians to Keynesians. The remaining taxed profits are transferred to Ricardian agents per equation (14).

### 2.7 Equilibrium

A competitive equilibrium in this economy is defined by two elements: 1) a sequence of stochastic processes $\{Y_t, Y_{rt}, Y_{kt}, C_{rt}, C_{kt}, N_t, R_t, \Pi_t, w_t, w_t^*, d_t, t_{rt}, t_{kt}, b_{rt}, b_{kt}\}_{t=0}^{\infty}$; and 2) an exogenous process $\{A_t\}_{t=0}^{\infty}$, such that Ricardian agents solve their maximization problem respecting budget constraint (1) and consumption Euler equation (2); Keynesian agents respect their budget constraint (4); income flows are defined by (3) and (5); labor supply is defined by (6) and (7); output is produced according to the aggregate production function (9) and dividends are defined by (10); inflation follows the New Keynesian Phillips curve (8); transfers are defined by (13) and (14); goods market clears (11); bond positions are such that $b_{kt} = \underline{b} = 0$ and the bonds market clears (12). The model is closed by specifying a rule for monetary policy. Monetary policy affects the equilibrium through the Euler equation of the Ricardian agent and through general equilibrium effects on wages and profits. In the next sections we consider two possibilities: 1) a central bank that pursues welfare-based optimal monetary policy, and 2) a central bank that pursues different interest rate rules (Taylor rules).

### 2.8 Approximate (Linearized) Equilibrium

Following the tradition of the New Keynesian literature, we log-linearize the equilibrium conditions around the deterministic steady state. For a variable $x_t$, whose steady state value is $\overline{x}$, we define $\hat{x}_t \equiv \ln x_t - \ln \overline{x}$.

[17] Note that we have already substituted the market clearing condition for labor $N_t = \int_0^1 L_{jt} dj = L_t$.

[18] This assumption allows us to ignore the distortions caused by monopolistic competition.

[19] This is consistent with Debortoli and Gali, 2018 and a departure from the assumption of different taxes in the steady-state and over the business cycle in Debortoli and Gali, 2017.

Log-linearized equilibrium conditions follow. We ignore the budget constraint of Ricardian agents due to Walras Law. Furthermore, we make use of the constant bond positions to disregard $b_{rt}$, $b_{kt}$, and to simplify $Y_{rt} = C_{rt}$, $Y_{kt} = C_{kt}$. An approximate equilibrium in this economy has two elements: 1) a sequence of stochastic processes $\left\{\hat{Y}_t, \hat{C}_{rt}, \hat{C}_{kt}, \hat{N}_t, \hat{R}_t, \ \hat{\Pi}_t, \hat{w}_t, \hat{w}^*_t, \hat{d}_t, \hat{t}_{rt}, \hat{t}_{kt}\right\}_{t=0}^{\infty}$; and 2) an exogenous process $\left\{\hat{A}_t\right\}_{t=0}^{\infty}$, such that given the initial state $\hat{w}_{-1}$, the following hold:[20]

$$\hat{C}_{rt} = E_t\hat{C}_{rt+1} - \hat{R}_t + E_t\hat{\Pi}_{t+1} \tag{15}$$

$$\frac{\overline{C}_k}{\overline{w}\overline{N}}\hat{C}_{kt} + \frac{T_p\overline{Y}}{\overline{w}\overline{N}}\hat{Y}_t - \frac{\overline{t}_k}{\overline{w}\overline{N}}\hat{t}_{kt} = -\gamma\hat{A}_t + \hat{w}_t + \hat{N}_t \tag{16}$$

$$\hat{w}^*_t = \phi\hat{N}_t + \hat{Y}_t \tag{17}$$

$$\hat{w}_t = (1-\psi_w)\,\hat{w}^*_t + \psi_w\left(\hat{w}_{t-1} - \hat{\Pi}_t\right) \tag{18}$$

$$\hat{Y}_t = \hat{A}_t + (1-\alpha)\,\hat{N}_t \tag{19}$$

$$(1+T_p)\,\hat{Y}_t = \frac{\overline{d}}{\overline{Y}}\hat{d}_t + \frac{\overline{w}\overline{N}}{\overline{Y}}\left(\hat{w}_t + \hat{N}_t\right) \tag{20}$$

$$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \frac{\theta_p}{\psi_p}\left(\hat{w}_t + \frac{\alpha}{1-\alpha}\hat{Y}_t - \frac{1}{1-\alpha}\hat{A}_t\right) \tag{21}$$

$$\hat{d}_t = \frac{(1-\lambda)\,\overline{t}_r}{\delta\overline{d}}\hat{t}_{rt} + \frac{\lambda\overline{t}_k}{\delta\overline{d}}\hat{t}_{kt} \tag{22}$$

$$\hat{t}_{kt} = \hat{d}_t \tag{23}$$

$$\hat{Y}_t = \lambda\frac{\overline{C}_k}{\overline{Y}}\hat{C}_{kt} + (1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\hat{C}_{rt} \tag{24}$$

and a specification for monetary policy.

## 2.9 Steady State

The steady state of this economy generally depends on structural parameters that i) determine monopolistic distortions and the offsetting subsidies ($T_p$ and $\theta_p$),[21] ii) govern fiscal redistribution ($\delta$, $\tau$), iii) characterize the production function ($\alpha, \overline{A}$), iv) define preferences ($\beta$, $\phi$), and v) characterize population shares ($\lambda$). Note that the steady state does not depend on $\gamma$ as it governs only the degree of "cyclical" skill-biasedeness.

## 2.10 Technology Shocks

Throughout the paper we assume that the economy starts in the steady state and then is exposed to a one time, unexpected shock $\epsilon_t$ that increases productivity $A_t$ at $t = 0$. Specifically, we assume that

$$A_t = \overline{A}e^{\epsilon_t},$$

[20] In the simulations, we set the initial real wage deviation $\hat{w}_{-1}$ to zero, i.e. we assume the economy starts from a state in which real wages are at their steady state level.

[21] We assume that $(1+T_p)\,\frac{\theta_p - 1}{\theta_p} = 1$.

which is linearized to

$$\hat{A}_t = \epsilon_t.$$

The productivity shock is deterministic and evolves according to $\epsilon_t = \rho\epsilon_{t-1}$, and $\epsilon_0 = 0.01$.

### 2.11 Calibration

The main parameters to be calibrated are $\lambda$, $\tau$, $\delta$, and $\gamma$. The rest are set to standard values from the literature.

We set the share of Keynesians in the population to be 40%, or $\lambda = 0.4$, which is in the middle of a wide range used in the literature, see Coenen et al., 2012. Since the profits tax and redistribution are isomorphic, we set $\delta = 1$ and we calibrate the redistribution parameter to $\tau = 0.93$ to match the ratio of non-labor income of households in the top 60 percent of the income distribution to the bottom 40 percent in the Survey of Consumer Finances for the U.S. in 2016.[22]

The parameter $\gamma$ is calibrated to match the effect of a productivity shock on consumption across the income distribution as estimated in De Giorgi and Gambetti, 2017. This is done assuming that monetary policy follows a Taylor rule with standard response parameters.[23]

Table 1 summarizes the calibration used henceforth. We set the profits share to 0.25. The discount factor (at the quarterly frequency) is set such that the steady state real interest rate is 3% on a yearly basis. Phillips curve parameters are taken from Debortoli and Gali, 2017, which implies a slope corresponding to an average price duration of 4 quarters in a world with Calvo rigidities. Consistent with empirical evidence from Barattieri et al., 2014 and the Bayesian estimates in Smets and Wouters, 2007, the wage rigidity parameter is set to $\psi_w = 0.75$.

Table 1: Calibration

| Parameter | Value | Concept | Source |
|---|---|---|---|
| $\alpha$ | 0.25 | Profits Share | Standard, e.g. Galí, 2015 |
| $\beta$ | 0.9925 | Discount Factor | Standard, e.g. Galí, 2015 |
| $\lambda$ | 0.4 | Share of Keynesian | Coenen et al., 2012 |
| $\chi$ | 1 | Disutility of Labor | Standard, e.g. Galí, 2015 |
| $\phi$ | 1 | Frisch Elasticity | Standard, e.g. Galí, 2015 |
| $\theta_p$ | 9 | Elasticity of Intermediate Goods | Standard, e.g. Galí, 2015 |
| $\psi_p$ | 372.8 | Rotemberg Cost | Debortoli and Gali, 2017 |
| $\psi_w$ | 0.75 | Wage Rigidity | Smets and Wouters, 2007 |
| $\rho$ | 0.9 | Persistence Of Tech Shock | Standard, e.g. Galí, 2015 |
| $\tau$ | 0.93 | Degree of Redistribution | Calibrated, see discussion above |
| $\gamma$ | 2.21 | Tech Bias | Calibrated, see discussion above |

[22] This ratio is 24.5. $\tau$ is chosen so that $\frac{\bar{t}_r}{\bar{t}_k} = 24.5$.

[23] We use $\phi_\pi = 1.5$ and $\phi_y = 0.125$ in a standard Taylor rule.

# 3 Optimal Monetary Policy

In this section, we show that a welfare-maximizing central bank caring equally about all agents in the economy described in Section 2 should place a non-zero weight on stabilizing consumption inequality and the labor income share, beyond the usual objectives of stabilizing inflation and output gaps. However, the optimal weight on consumption inequality under the optimal policy is relatively small compared to those on inflation and output gaps. Interestingly, the optimal weight on the labor income share rises with the degree of inequality in steady state, while that on inflation falls, meaning stabilizing inflation is less appealing the larger the steady state inequality. Still, the magnitude of the welfare weights is not sufficient to judge whether these two additional objectives are important. To assess the welfare gains from targeting consumption inequality and the labor income share, we compare the welfare achieved by a central bank pursuing fully optimal policy with that achieved by a central bank that chooses to only stabilize inflation and output gaps. Such a central bank conducts what we refer to as a "Representative Agent New Keynesian (RANK) optimal policy" knowing that there is inequality in the economy and its implications, but not including either consumption inequality or labor share in its objective function. The policy pursued by the representative-agent central bank only entails small welfare losses relative to the fully optimal policy that incorporates inequality because the weight on inflation is much larger than all others and thus preventing inflation volatility is still first order important even when there is substantial steady-state inequality. Note that when there is no steady state inequality, optimal monetary collapses to the typical RANK optimal policy.[24]

In what follows, we derive the Ramsey social welfare function, we define and discuss the resulting optimal monetary policy, we define a so-called RANK-optimal policy, and finally we compare the optimal policy with this RANK-optimal policy under technology shocks.

## 3.1 The Social Welfare Function

We consider the lifetime utility of all agents in this economy as a measure of aggregate welfare:

$$\mathbf{W} \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[ \lambda \ln C_{kt} + (1-\lambda) \ln C_{rt} - \frac{N_t^{1+\phi}}{1+\phi} \right]. \tag{25}$$

This measure of welfare is utilitarian — one Ricardian agent is valued equally as one Keynesian agent.[25] Following the optimal policy literature, we take a second order approximation to the welfare function de-

[24] This is corresponds to Debortoli and Gali, 2017 in the case of $\chi_n = 0$, because in our model we assume profits are taxed at a single rate over the business cycle and in steady-state.

[25] $\lambda$ accounts for the possibility that one type of agent is more prevalent than the other.

scribed in equation (25) and re-write it as a discounted sum of four quadratic gaps:[26]

$$\mathbf{W} \approx -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ W_\Pi \hat{\Pi}_t^2 + W_Y (\hat{Y}_t - \hat{A}_t)^2 + W_{\Delta_c} (\hat{C}_{rt} - \hat{C}_{kt})^2 + W_{LS} (\widehat{LS}_t - \widehat{LS}_t^*)^2 \right\} + T_0 + t.i.p.. \quad (26)$$

where:

$$W_\Pi = \psi_p - \psi_p \Psi(\tau) \left(1 - \frac{\overline{d}}{\overline{\overline{Y}}}\right) \quad (27)$$

$$W_Y = \frac{1+\phi}{1-\alpha} \quad (28)$$

$$W_{\Delta_c} = \lambda \left(1 - \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right) \left(1 - \lambda \left(1 - \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right)\right) \quad (29)$$

$$W_{LS} = (1-\alpha)^2 \frac{\Psi(\tau)^2}{\lambda \tau \frac{\overline{d}}{\overline{\overline{Y}}}} \quad (30)$$

and $\hat{LS}_t = \hat{w}_t + \hat{N}_t - \hat{Y}_t$ is the labor share and $\widehat{LS}_t^*$ is its welfare relevant counterpart.[27]

This welfare function consists of four gaps: the standard inflation gap, $\hat{\Pi}_t$, and output gap, $\hat{Y}_t - \hat{A}_t$, the consumption inequality gap, $\hat{C}_{rt} - \hat{C}_{kt}$, defined as the change in the consumption of the Ricardian relative to the Keynesian agent, and the deviation of the labor share from its welfare-relevant target, $\widehat{LS}_t - \widehat{LS}_t^*$. $T_0$ is a predetermined function of only gaps at time zero. $t.i.p.$ stands for terms independent of (monetary) policy.[28]

## 3.2 Intuition for the Welfare Weights

To provide intuition on the weights in the loss function in equation (26), the following second order expansion of the Ramsey social utility function (equation (25)) is a useful intermediate representation:

$$\mathbf{W} \approx E_0 \sum_{t=0}^{\infty} \beta^t \left\{ -\lambda \tau \frac{\overline{d}}{\overline{\overline{Y}}} (\hat{C}_{rt} - \hat{C}_{kt}) - \frac{\psi_p}{2} \hat{\Pi}_t^2 - \frac{1}{2} W_Y (\hat{Y}_t - \hat{A}_t)^2 - \frac{1}{2} W_{\Delta_c} (\hat{C}_{rt} - \hat{C}_{kt})^2 \right\} + t.i.p.. \quad (31)$$

Equation (31) generalizes two cases already studied in the literature. The first is the standard textbook RANK welfare function, which can be obtained by either assuming away inequality in the steady state ($\tau = 0$) and in the dynamics around the steady state ($\hat{C}_{kt} = \hat{C}_{rt}$), or by assuming there are no Keynesian agents ($\lambda = 0$). In either case, the linear and the quadratic terms on consumption inequality drop out and we are left with the familiar quadratic terms on inflation and output gap deviations from the standard RANK model. The second case is the welfare function derived in Debortoli and Gali, 2017, which can be obtained by assuming the steady state is efficient ($\tau = 0$), there are Keynesian agents in the

[26] Simplifying (25) into a quadratic loss function involves two steps. First, we take a second-order expansion following Rotemberg and Woodford, 1997 and Woodford, 2002. In many applications, such a second-order expansion leads immediately to a simple quadratic loss function because the steady state is efficient. For example, this is the case in Debortoli and Gali, 2017, who consider $\tau = 0$. In our economy though, the steady-state is inefficient and the second-order expansion of equation (25) thus includes linear terms on the labor share gap. A second step involves adapting the method in Benigno and Woodford, 2005 and deriving the objective function under "timeless commitment", resulting in a loss function that is quadratic up to a term that only depends on gaps at time zero.

[27] See Appendix A.1-A.4 for detailed derivations, the analytical expression for each weight, the definition of welfare-relevant labor share.

[28] See Appendix A.4 for the expression of $T_0$.

economy ($\lambda > 0$), and inequality over the business cycle ($\hat{C}_{kt} \neq \hat{C}_{rt}$). Under these assumptions, the linear term on consumption inequality drops out and the quadratic term on consumption inequality remains, with a simplified weight of $\lambda(1-\lambda)$, representing welfare losses stemming exclusively from business cycle fluctuations in inequality.

Equation (31) importantly includes a new — compared to a model without steady-state inequality — linear term on changes in consumption inequality that governs the importance of first order welfare gains in reducing inequality. These gains stem from the concavity of the utility function. Because the term is negative and linear, welfare declines as consumption inequality rises. The marginal utility of Keynesians is always larger than that of Ricardians, meaning that the benefit of higher consumption for the Ricardian does not outweigh the loss from taking away from the Keynesian. This linear term, $-\lambda\tau\frac{\overline{d}}{\overline{Y}}$, has three components, with welfare declining with the share of Keynesians ($\lambda$), steady-state inequality ($\tau$) or as dividends take up a greater share of income ($\frac{\overline{d}}{\overline{Y}}$). Intuitively, the social planner will care more about this term if there are more Keynesians, particularly if they are poorer on average and/or if Ricardians are taking a greater share of income because of their preferential access to dividends.

This linear term has implications for the ultimate welfare weights on inflation and labor share gaps in equation (26), making them depend on the degree of steady-state inequality as governed by $\tau$. The weight on consumption inequality also depends on steady state inequality, but not due to this linear term.

We now turn to providing intuition as to how the welfare weights in equation (26) depend on the degree of inequality in steady-state.[29] Of the three weights on inflation gap, labor share gap, and consumption inequality gap, only the latter is directly visible in the intermediate representation of the welfare function in equation (31). We thus discuss that weight first.

The welfare weight on consumption inequality is:

$$W_{\Delta_c} = \lambda \underbrace{\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)}_{=\overline{C}_k/\overline{Y}} \left(1 - \lambda \underbrace{\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)}_{=\overline{C}_k/\overline{Y}}\right)$$

This weight is small under standard calibrations of the labor share and price rigidities. Intuitively, aggregate fluctuations affect all agents proportionally, while fluctuations in consumption inequality are of second-order importance and disproportionately affect one type of agent. The weight depends on the interaction between the fraction of Keynesian agents, $\lambda$, and the steady-state consumption of the average Keynesian agent, which depends on $\tau$. The weight has an inverted U-shape, defined between zero and one, and is maximized when the consumption shares of Keynesians and Ricardians are equalized, i.e. $\lambda\frac{\overline{C}_k}{\overline{Y}} = (1-\lambda)\frac{\overline{C}_r}{\overline{Y}}$. This is because the planner places more weight on the consumption inequality gap if the two groups are similar in size. As steady state inequality rises, and if Keynesians form a small minority (as in our calibration where $\lambda$ is less than half), the weight on consumption inequality falls because it is increasingly costly to redistribute in favor of a minority at the expense of a majority already enjoying high consumption.

[29] The weight on output gap is not discussed as it does not depend on $\tau$ or $\lambda$ and is the same as under the RANK economy.

The weight on inflation is given by the following expression:

$$W_{\Pi} = \psi_p \left[1 - \Psi(\tau)\left(1 - \frac{\overline{d}}{\overline{\overline{Y}}}\right)\right]$$

where $\Psi(\tau)$ is defined in equation (60). The weight on inflation above differs from that in a RANK economy as long as $\Psi(\tau) \neq 0$, which would be the case if either $\lambda = 0$ or $\tau = 0$.

As steady-state inequality rises, or $\tau$ increases, the weight on inflation falls. This can be shown analytically as $\Psi(\tau)$ is trivially rising in $\tau$ (See Appendix A.4). Intuitively, there are two channels at work. First, inflation volatility takes resources away from the aggregate (as can be seen from the aggregate resource constraint in equation (11)). For a given level of Keynesian agent's consumption, inflation volatility lowers the consumption of the Ricardian agent, thereby reducing inequality. Second, since dividends decline with inflation volatility due to the existence of Rotemberg adjustment costs, inflation volatility also affects inequality through its differential effects on the budget constraints, and hence consumption, of each agent. Thus, the higher the $\tau$, or the less dividends are distributed, the less inflation will affect the consumption of the Keynesian. Given the redistributional role of inflation volatility in this model, the central bank is thus more willing to tolerate fluctuations in inflation the higher the steady-state inequality.

Turning to the weight on the labor share, defined by:

$$W_{LS} = (1-\alpha)^2 \frac{\Psi(\tau)^2}{\lambda\tau\frac{\overline{d}}{\overline{\overline{Y}}}}$$

Conversely, the weight on the labor share rises as steady-state inequality increases. Analytically, it can be shown that $\frac{\Psi(\tau)^2}{\lambda\tau\frac{\overline{d}}{\overline{\overline{Y}}}}$ is rising with $\tau$ (See Appendix A.4). Intuitively, the labor share affects inequality through the exposure of different households to fluctuations in income sources. In particular, a higher value of $\tau$ means that Keynesians are more reliant on labor income. This implies that labor income can play a larger role in closing the inequality gap as $\tau$ rises, which translates into a higher corresponding welfare weight. Note that the weight on the labor share would be zero in a steady-state with no inequality, since under that case $\Psi(0) = 0$, unlike the weight on inflation which just collapses to that under the RANK economy.

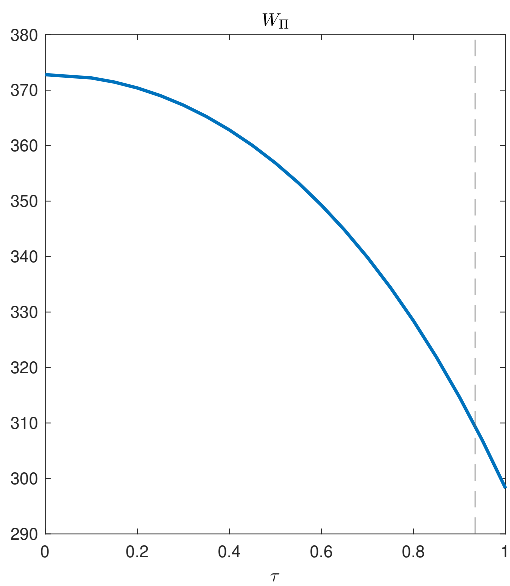
Figure 1 presents the weights on the four gaps, $W_{\Pi}, W_Y, W_{\Delta_c}$ and $W_{LS}$, in equation (26) depending on the degree of steady state inequality in this economy as governed by $\tau$ under our calibration.[30] Three observations can be gleaned from Figure 1: (i) the weight on consumption inequality is not zero but it is small compared to that on the output gap, which itself is much smaller than that placed on the inflation gap;[31] (ii) the weight on labor share is also not zero in our baseline calibration but it is also relatively small; and finally (iii) the weight on inflation gap is the largest but declines with $\tau$,[32] while the weight on the labor share gap rises with $\tau$, and is zero for the economy without steady state inequality.

[30]The welfare weights do not depend on $\gamma$, the degree of cyclical skill-bias in wages, because the weights themselves are evaluated in the steady state where the skill-bias is immaterial by assumption. However, $\gamma$ affects welfare directly through its impact on the welfare relevant measure of the labor income share and indirectly through its impact on the dynamics of the economy. Appendix A.4 derives the analytical expressions for the weights.

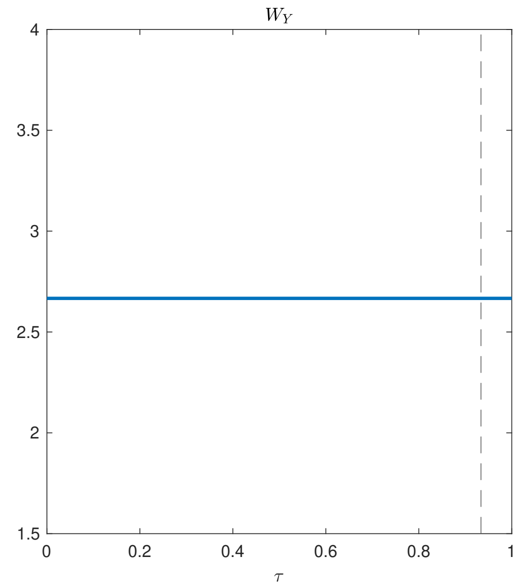
[31]At our calibrated values of $\tau$ and $\gamma$, the weights on inflation, output, consumption inequality and labor share gaps are 310, 2.7, 0.2, and 0.3, respectively. In our calibration, $\lambda$ is smaller than one half, the critical threshold discussed above. Thus, the weight on consumption inequality declines with $\tau$.

[32]Specifically, the weight on inflation gap declines by 17 percent from the egalitarian society, $\tau = 0$, relative to that in our baseline calibration, $\tau = 0.93$.

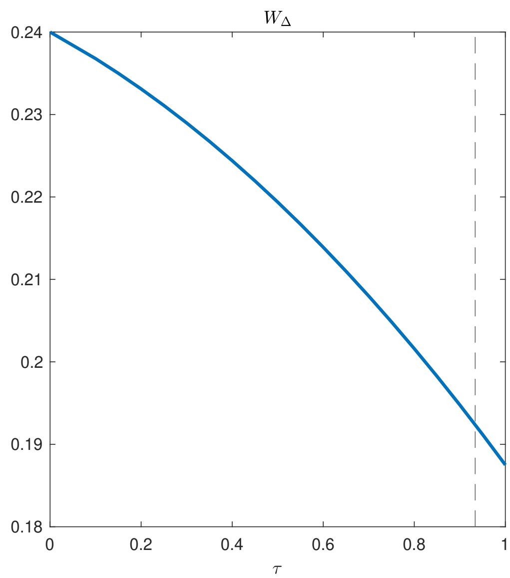
Figure 1: Welfare Weights and Redistribution Parameter $\tau$



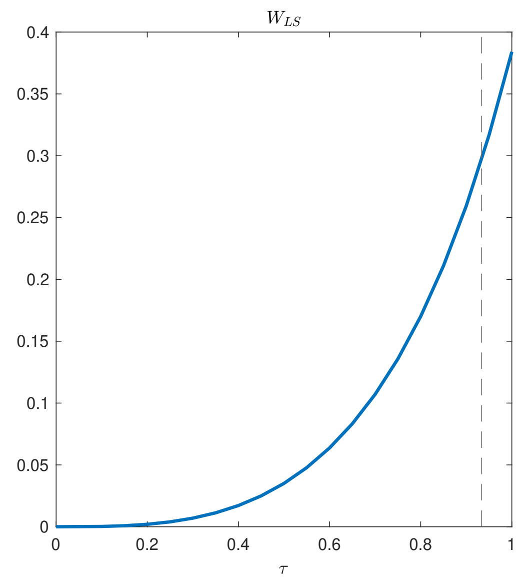

(a) Weight on Inflation Gap




(b) Weight on Output Gap




(c) Weight on Consumption Inequality Gap




(d) Weight on Labor Share Gap

*Notes:* The panels show the four weights as a function of the extent to which dividends are redistributed, governed by $\tau$. All other parameters are set according to the baseline calibration in Table 1.

## 3.3 Optimal Monetary Policy

Having established the social welfare function, we now turn to policy evaluations. In what follows, we explain the main experiment and policy comparison that we pursue to understand the welfare implications of inequality for the conduct of optimal policy.

Firstly, we evaluate "Optimal Policy", defined as the solution to the Ramsey problem of a central bank that aims to maximize the welfare function described by equation (26), subject to the structural equations (15)-(24). Appendix A.5 reports the Lagrangian of the problem whose optimality conditions are:

$$-W_{\Pi}\hat{\Pi}_t + \mu_{1t} - \mu_{1t-1} + \psi_w \mu_{4t} = 0 \tag{32}$$

$$-W_Y(\hat{Y}_t - \hat{A}_t) - \frac{\alpha}{1-\alpha}\mu_{2t} - (1-\psi_w)\frac{1-\alpha+\phi}{1-\alpha}\mu_{4t} = 0 \tag{33}$$

$$-W_{\Delta_c}\left(\hat{C}_{rt} - \hat{C}_{kt}\right) - \left(1-\tilde{\lambda}\right)\mu_{3t} = 0 \tag{34}$$

$$-W_{LS}\left(\hat{LS}_t - \hat{LS}_t^*\right) - \frac{\theta_p}{\psi_p}\mu_{1t} + \mu_{2t} - \frac{(1-\alpha)\tau}{\frac{\overline{C}_k}{\overline{Y}}}\mu_{3t} = 0 \tag{35}$$

$$-\mu_{2t} + \mu_{4t} - \beta\psi_w E_t \mu_{4t+1} = 0 \tag{36}$$

where $\mu_i$ are Lagrange multipliers of the Ramsey problem.

It can be shown that optimal monetary policy is characterized by the following price level targeting condition:[33]

$$\hat{P}_t = -\Theta_{LS}\left(\hat{LS}_t - \hat{LS}_t^*\right) + \Theta_{\Delta_c}\left(\hat{C}_{rt} - \hat{C}_{kt}\right) - (\Theta_1 - \Theta_2)\left(\hat{Y}_t - \hat{A}_t\right) + \Theta_2 E_t \sum_{j=0}^{t}\left(\hat{Y}_j - \hat{A}_j\right) +$$
$$- \Theta_2\left(1 - \Theta_3 - \frac{\Theta_4}{\Theta_2}\right)\left[\sum_{j=0}^{t} E_t \sum_{s=0}^{\infty}\Theta_3^s\left(\hat{Y}_{j+s} - \hat{A}_{j+s}\right)\right] \tag{37}$$

where

$$\Theta_{LS} = \frac{1}{W_\Pi}\frac{\psi_p}{\theta_p}W_{LS} \qquad \Theta_{\Delta_c} = \frac{1}{W_\Pi}\frac{\psi_p}{\theta_p}\frac{(1-\alpha)}{1-\tau\frac{\overline{d}}{\overline{Y}}}\frac{1}{\frac{\overline{d}}{\overline{Y}}}W_{\Delta_c}$$

$$\Theta_1 = \frac{1}{W_\Pi}\frac{\psi_p}{\theta_p}\frac{1-\alpha}{\alpha}W_Y \qquad \Theta_2 = \frac{1}{W_\Pi}\frac{\psi_p}{\theta_p}\frac{1-\alpha}{\alpha}\frac{(1-\psi_w)(1-\alpha+\phi)}{\alpha+(1-\psi_w)(1-\alpha+\phi)}W_Y$$

$$\Theta_3 = \frac{\alpha\beta\psi_w}{\alpha+(1-\psi_w)(1-\alpha+\phi)} \qquad \Theta_4 = \frac{1}{W_\Pi}\frac{\psi_w(1-\alpha)}{\alpha+(1-\psi_w)(1-\alpha+\phi)}W_Y$$

Equation (37) implies that a central bank that follows the optimal monetary policy is willing to tolerate larger price fluctuations if: 1) the labor share is smaller; 2) inequality is higher; 3) the output gap is smaller; 4) past output gaps were larger; and 5) future (past) output gaps are (were) smaller. It is not possible to stabilize all four gaps. Consider an extreme case in which the central bank is able to stabilize inflation, output, and labor share gaps. Because of tech bias, the ensuing relative income fluctuations will inevitably lead to consumption inequality. As steady-state inequality rises ($\tau$ increases), $W_\Pi$ falls, leading the central bank to be more willing to accept fluctuations in the price level vis-a-vis other business cycle

[33] See Appendix A.6 for the derivations.

variables.

Past and future output gaps enter equation (37) because the central bank internalizes that inflation has a persistent effect on future real wages, and hence output, due to wage rigidities.[34]

### 3.4 The RANK Optimal Monetary Policy

We now introduce a "RANK-optimal" policy to assess the relative benefits of stabilizing inequality and labor share gaps, beyond the usual stabilization of inflation and output gaps. Under this policy the central bank decides not to attach any weight to inequality, and thereby act as if the economy has a representative agent, despite being aware of the existence of inequality. In other words, RANK-optimal policy focuses on maximizing the average consumption and concerns itself solely with stabilizing inflation and output gaps.[35] Taking this fact into account leads to the following price level targeting rule:[36]

$$\frac{\psi_p}{W_\Pi}\hat{P}_t = -\left(\Theta_1-\Theta_2\right)\left(\hat{Y}_t-\hat{A}_t\right)+\Theta_2 E_t\sum_{j=0}^{t}\left(\hat{Y}_j-\hat{A}_j\right)-\Theta_2\left(1-\Theta_3-\frac{\Theta_4}{\Theta_2}\right)\left[\sum_{j=0}^{t}E_t\sum_{s=0}^{\infty}\Theta_3^s\left(\hat{Y}_{j+s}-\hat{A}_{j+s}\right)\right] \tag{39}$$

By comparing the fully optimal and RANK-optimal policies, we can assess how introducing non-zero weights on consumption inequality and labor share gap in the central bank's objective function changes the conduct of monetary policy and how it affects welfare. The RANK-optimal policy is also a useful benchmark to consider given the extensive literature on optimal policy in economies with a representative agent.

### 3.5 Responses to TFP shocks

Figure 2 shows impulse responses of key gaps to a positive TFP shock under optimal and RANK-optimal policies, as defined by equations (37) and (39).

Optimal policy reduces the volatility of both consumption inequality and the labor share gap more forcefully than RANK-optimal policy (Panels d and e, respectively). In doing so, optimal policy achieves

[34] This can be seen by considering the special case when wages are flexible, i.e. $\psi_w = 0$, where equation (37) simplifies to the following price level targeting rule:

$$\hat{P}_t = -\Theta_{LS}\left(\hat{LS}_t-\hat{LS}_t^*\right)+\Theta_{\Delta_c}\left(\hat{C}_{rt}-\hat{C}_{kt}\right)-\left(\Theta_1-\Theta_2\right)\left(\hat{Y}_t-\hat{A}_t\right) \tag{38}$$

Equation (38) generalizes two cases already studied in the literature. The first is the textbook RANK problem under flexible wages and no steady-state inequality ($\lambda=\tau=\psi_w=0$). Under that economy, monetary policy can fully stabilize inflation and output gaps in the presence of technology shocks and the optimality condition boils down to a simple trade-off between the two gaps. For such a RANK economy, $\Theta_{LS}=\Theta_{\Delta_c}=\Theta_3=\Theta_4=0$ and $\Theta_1-\Theta_2=\frac{1}{\theta_p}$, see equation 5.14 in Galí, 2015. Debortoli and Gali, 2017 study a second particular case where there is no inequality, either in the steady state or over the business cycle ($\tau=\psi_w=0$, $\lambda>0$). It follows that $\Theta_{\Delta_c}=0$ in equation (38), which then boils down to the unnumbered equation in page 31 of Debortoli and Gali, 2017 if $\chi_y=0$. In that economy, the central bank is again able to fully stabilize inflation and output gaps. Another useful case to consider is where there is a representative agent and wages are fully rigid (i.e. $\lambda=\tau=0$ and $\psi_w=1$). Under that case, equation (37) simplifies to the following price level targeting rule:

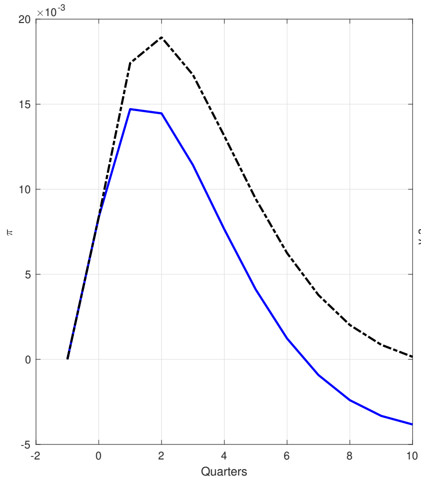
$$\hat{P}_t = -\Theta_1\left(\hat{Y}_t-\hat{A}_t\right)+\Theta_4\sum_{j=0}^{t}E_t\sum_{s=0}^{\infty}\beta^s\left(\hat{Y}_{j+s}-\hat{A}_{j+s}\right)$$

.

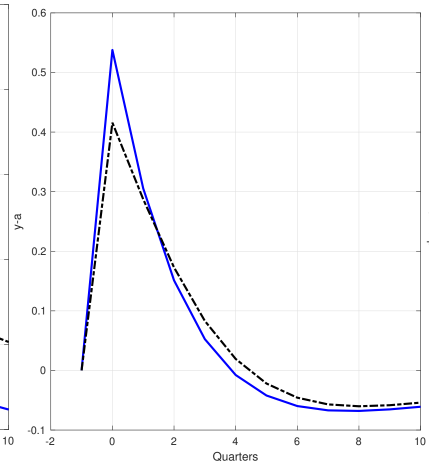
[35] In this problem, the objective function of "RANK-optimal" policy corresponds to the textbook welfare criterion in a representative agent model: $\mathbf{W_{RANK}} \approx \sum_{t=0}^{\infty}\beta^t\left\{-\frac{1}{2}W_{RANK,\Pi}\hat{\Pi}_t^2-\frac{1}{2}W_{RANK,Y}\left(\hat{Y}_t-\hat{A}_t\right)^2\right\}+T_0+t.i.p.$. See Galí, 2015.

[36] In what follows, $T_0$ is constrained to be the same value attained by optimal policy to ease comparison.

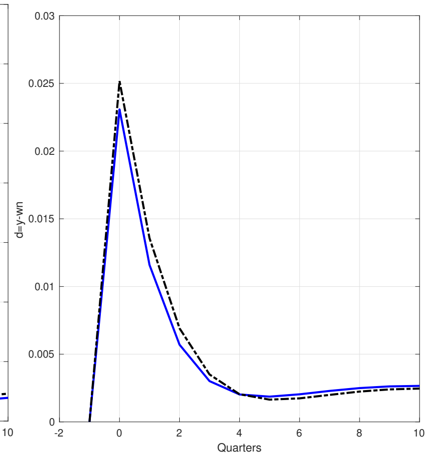
Figure 2: Impulse Response to a Positive TFP shock: Optimal vs RANK-Optimal Policy



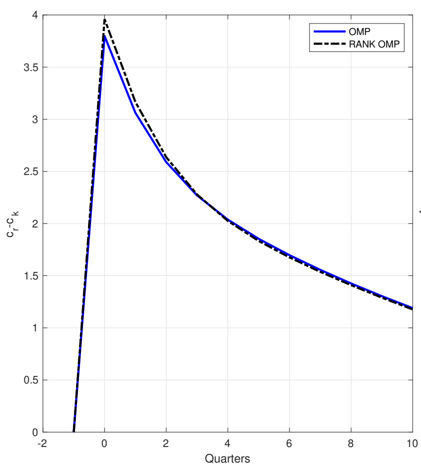

(a) Inflation Gap



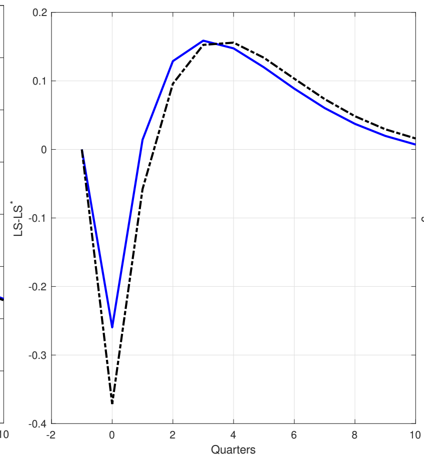

(b) Output Gap



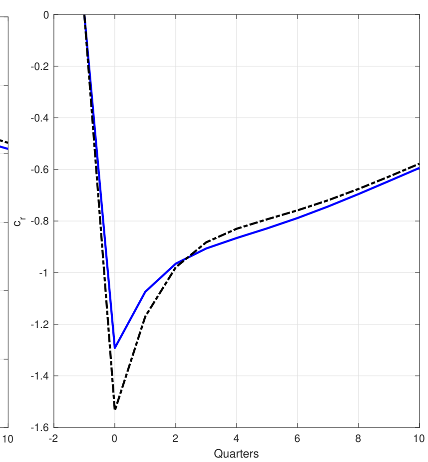

(c) Dividends



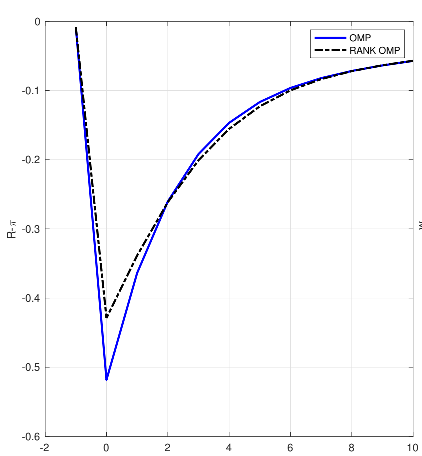

(d) Consumption Inequality Gap



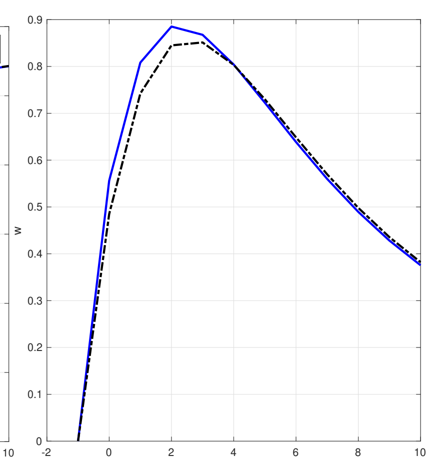

(e) Labor Share Gap



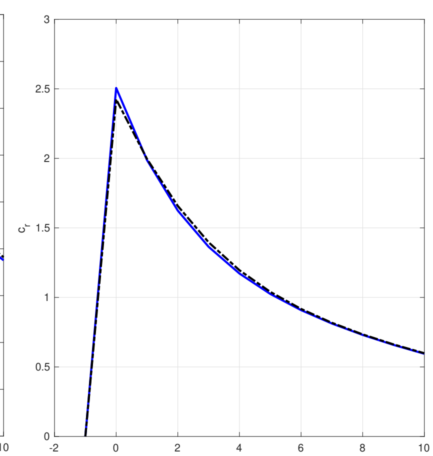

(f) Consumption of Keynesian




(g) Real Interest Rate



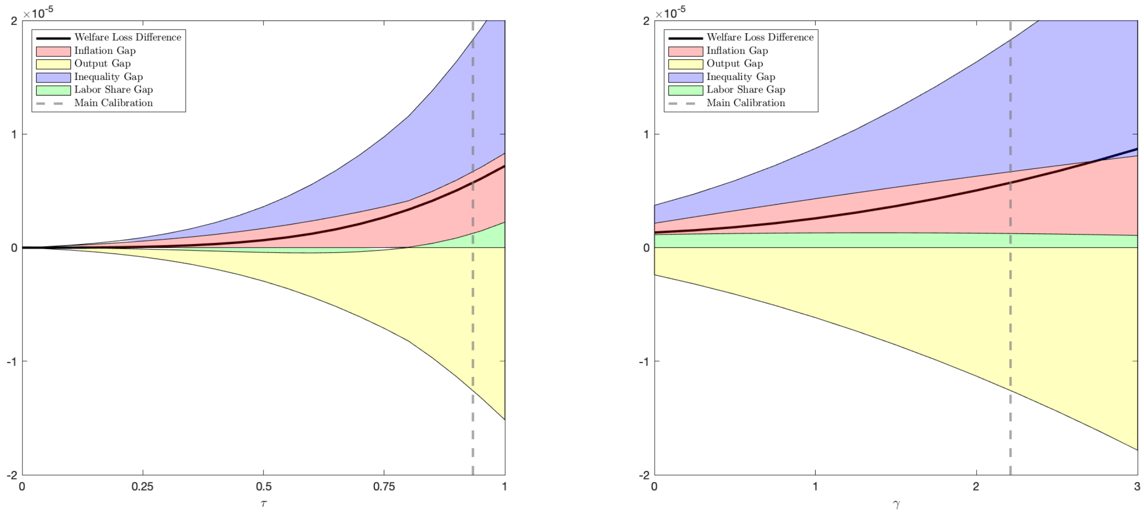

(h) Wage Rate




(i) Consumption of Ricardian

*Notes:* The panels show impulse response functions of inflation, output gap, dividends, consumption inequality, labor share gap, consumption of Keynesian agent, real interest rate , wage rate, and consumption of Ricardian agent, respectively, in percentage points under the fully optimal policy (blue) and the RANK-optimal policy (black dash-dotted). The y-axis measures the deviation from each variable's steady state, in percentage points.

Figure 3: Welfare Gain of Moving to Fully Optimal Policy from RANK-Optimal Policy, in Consumption Equivalent Terms




*Notes:* The panels show the difference in welfare between Optimal Policy and Rank-Optimal Policy as functions of the underlying parameters governing inequality in the economy. Shaded areas represent the differences by gaps (inflation, output, inequality, and labor share). The x-axis on the left (right) panel represents $\tau$ ($\gamma$). The vertical dashed line denotes our baseline calibration.

a more positive output gap (Panel b). The more positive output gap reflects a more aggressive monetary policy accommodation, which causes reallocation towards the Keynesian consumer compared to under the RANK-optimal policy. This reallocation happens through a relatively smaller increase in dividends (Panel c), and a relatively larger increase in wages (Panel h).[37]

The differences in impulse responses are not large, and hence the welfare achieved by either of the two policies is similar. Figure 3 shows the welfare loss in consumption-equivalent terms (black line), using (26) under each of these 2 policies, depending on $\tau$ and $\gamma$ and decomposing losses into those stemming from each of the four gaps (4 colored areas).[38] The black line is increasing in both $\tau$ and $\gamma$, which means optimal policy is increasingly superior to RANK-optimal policy as the underlying inequality rises. Under our calibration, the welfare gain is only about 0.5 x $10^{-6}$ percent in yearly consumption-equivalent terms for a 1 percent productivity shock, which is approximately 1 standard deviation. In relative terms, this means that moving to fully optimal policy from RANK-optimal policy only increases welfare by 1.2 percent.

Optimal policy achieves higher aggregate welfare relative to RANK-optimal policy primarily through reduced consumption inequality. This is seen by observing that the purple area is the largest positive in Figure 3. At the same time, there are also gains from a less volatile inflation gap (the red area) and labor share gap (green area) under our calibration, at the expense of more volatile output gap. In response to a

[37] A reader may ask why the inflation gap is smaller under optimal monetary policy than RANK optimal policy. This result can be gleaned from studying the Phillips Curve in equation (21). As initial inflation is similar in period 0 under both policies a larger wage- and output-gap (along with a similar productivity gap) under optimal monetary policy deliver a lower path for the inflation gap.

[38] We compute such compensation in percent of steady-state consumption of the average agent, as in Ravenna and Walsh, 2011. Specifically, we compute $x$ as the consumption equivalent representing the permanent increase in consumption that would increase welfare by the amount considered. Consider two monetary policies, $a$ and $b$, with the former being welfare-superior to the latter (i.e. $W_a > W_b$). We compute $x$ from the following formula $x = exp\left[(1-\beta)(W_a - W_b)\right] - 1$.

positive (negative) technology shock, optimal policy achieves higher (lower) real wages also through limiting the rise (fall) in inflation, and thereby ultimately limiting the rise (fall) of consumption inequality and the fall (rise) in the labor share.

The gains shown in Figure 3 come primarily from the existence of the consumption inequality term in the welfare function, i.e. a non-zero weight ($W_{\Delta_c}$) in equation 26. Appendix Figure B1 shows that a small departure from the RANK-optimal policy that includes the optimal weight on consumption inequality in the RANK welfare function (see footnote 34) is very close to fully optimal policy across $\tau$ and $\gamma$.

# 4 Taylor rules

Optimal monetary policy is hard to implement in practice. First, it requires extensive information about the structure of the economy including the paths of all variables under different scenarios. Second, precommitting to the full paths of all four gaps may not be credible. For these reasons policy makers sometimes make reference to simpler monetary policy rules.[39] A famous example of such a rule is the one proposed by Taylor, 1993.

In this section, we study augmented "Taylor Rules", that target either the consumption inequality gap and/or the labor share gap, in addition to the more traditional targeting of inflation and output gaps. We show that central banks using such augmented rules set lower rates than they would otherwise in response to a positive TFP shock. The mechanism is simple: a policy of lower interest rates leads to higher wages (and lower profits) on the margin, and thereby benefits disproportionately the poor, who rely more on labor income. Such a policy is beneficial not only because it lowers inequality, but also because it improves inflation and growth outcomes by avoiding an excessive tightening of the interest rate in response to a positive productivity shock. Interestingly, targeting the labor share alone seems sufficient to achieve the majority of the welfare gains. This is likely also easier to implement relative to targeting consumption inequality which requires more information. Augmented Taylor rules targeting the labor share gap deliver higher welfare ex-ante more generally, when cost-push and demand shocks are considered in addition to the technology shock. Moreover, these gains are robust to the underlying magnitude of inequality unlike the gains of targeting consumption inequality, which are very sensitive to the particular extent of inequality present in the economy. When there is no steady-state inequality, targeting the labor share performs similarly to the already well performing standard Taylor rule.

In the following three subsections, we discuss Taylor rules in three stages. Firstly, we evaluate the implications of the existence of inequality for standard Taylor rules in the presence of technology shocks without considering explicit inequality targeting. Secondly, we assess whether standard rules should be augmented given the presence of inequality and following technology shocks. Finally, we evaluate such augmented Taylor rules more broadly, for a wider constellation of typical business cycle shocks.[40]

[39] See for example https://www.federalreserve.gov/monetarypolicy/policy-rules-and-how-policymakers-use-them.htm and https://www.brookings.edu/blog/ben-bernanke/2015/04/28/the-taylor-rule-a-benchmark-for-monetary-policy/

[40] This exercise is similar to Ma and Park, 2021, that study central bank reactions to fluctuations in the income Gini index.

## 4.1 Standard Taylor Rules and Inequality

In this subsection, we consider traditional Taylor rules of the form:

$$\hat{R}_t = \phi_\pi \hat{\Pi}_t + \phi_y \left(\hat{Y}_t - \hat{A}_t\right) \tag{40}$$

where $\hat{R}_t$ is the deviation of the nominal interest rate from steady state, $\hat{Y}_t - \hat{A}_t$ is the output gap[41] and $\hat{\Pi}_t$ is the deviation of inflation from target.

Table 2 compares different Taylor rules of the form defined in equation (40). It begins by evaluating a typical rule with $\phi_\pi = 1.5$ and $\phi_y = 0.125$ as in Galí, 2015, Chapter 3. It then contrasts such a rule with three alternatives: (i) one with a zero output gap coefficient and $\phi_\pi = 1.5$; (ii) another where the coefficient on the output gap is one and $\phi_\pi = 1.5$; and (iii) finally, a rule with a coefficient on inflation of 5 and no coefficient on the output gap.

Table 2: Welfare in Taylor Rules - Standard Parametrization

| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
|---|---|---|---|---|---|---|---|---|
| $\tau$ | 0 | 0 | 0 | 0 | 0.93 | 0.93 | 0.93 | 0.93 |
| $\gamma$ | 0 | 0 | 0 | 0 | 2.21 | 2.21 | 2.21 | 2.21 |
| $\phi_\pi$ | 1.5 | 1.5 | 5 | 1.5 | 1.5 | 1.5 | 5 | 1.5 |
| $\phi_y$ | 0.125 | 0 | 0 | 1 | 0.125 | 0 | 0 | 1 |
| Cons. Equiv. Loss | 0.1 | 0.1 | 0 | 0 | 1.5 | 2 | 0.5 | 0.6 |
| Cons. Equiv. $W - T_0$ | 0.1 | 0.1 | 0 | 0 | 0.3 | 0.5 | 0.1 | 0.1 |
| Inflation | 94.5 | 94.7 | 91.1 | 94.7 | 65.1 | 75 | 23.5 | 20.9 |
| Output | 5.5 | 5.3 | 8.9 | 5.3 | 4.5 | 4.8 | 1.6 | 1.8 |
| Inequality | 0 | 0 | 0 | 0 | 28.3 | 18.5 | 72.6 | 74.3 |
| Labor Share | 0 | 0 | 0 | 0 | 2.1 | 1.8 | 2.3 | 3 |

*Notes*: "Cons. Equiv. Loss" is the loss in yearly consumption multiplied by 10000 (e.g. 1 means 0.0001 percent of steady state consumption); "Cons. Equiv. $W - T_0$" is the dynamically relevant welfare component in the same units as the "Cons. Equiv. Loss"; "Inflation" is the share of welfare loss that is due to a non-zero inflation gap; "Output" is the share of welfare loss that is due to a non-zero output gap; "Inequality" is the share of welfare loss that is due to a non-zero inequality gap; and "Labor Share" is the share of welfare loss that is due to a non-zero labor share gap.

In an economy without inequality (table 2, columns (1)-(4), when $\tau = \gamma = 0$) three conclusions emerge: (i) placing some weight on the output gap can be welfare improving, or divine coincidence does not hold, something that is well known for the case of wage rigidities (see discussion in Blanchard and Galí, 2007); (ii) a larger weight on inflation fares even better; and (iii) any welfare losses are disproportionately due to inflation gaps. These are all well understood in the literature (see for example Galí, 2015, Chapter 3) and reflect the disproportionately large weight on inflation in the social welfare function (equation (26)).

When inequality is introduced (table 2, columns (5)-(8)), the same three observations above continue to hold. Importantly, a higher parameter on inflation reduces total welfare losses from all four gaps.[42] Note, that the welfare loss due to the consumption inequality gap can be sizeable. It is always larger than

[41] An alternative is to target actual output. If output, and not output gap, is targeted, there is an additional benefit to targeting consumption inequality, or the labor share, because doing so allows to better proxy for the interest rate that would best respond to the output gap.

[42] This can be seen by comparing consumption equivalent losses across different Taylor rules. For instance, consider the welfare loss under column 6, which is 2. This is larger than 0.5 which is the welfare loss from targeting inflation more forcefully under column 7.

that from the output gap or the labor share gap. It is even larger than that stemming from inflation gaps when the Taylor rule reacts strongly to inflation (column 7) or when the central bank reacts more strongly to output gaps (column 8).

Table 2 suggests that aggressively responding to inflation gaps is a superior policy. Still, it is possible that welfare can be improved by allowing a small weight on ether consumption inequality gap or labor share gap. We study that possibility in what follows.

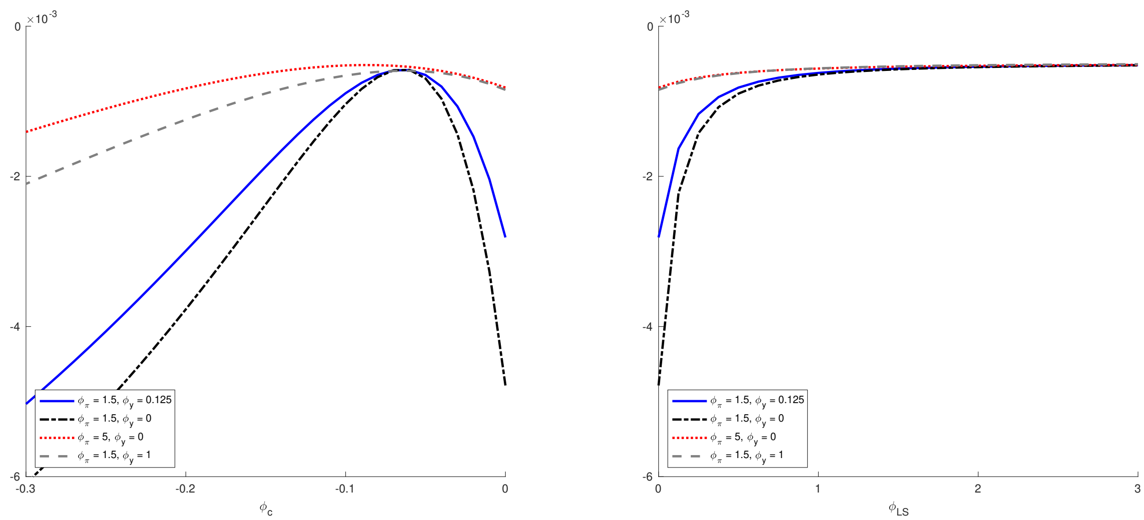
## 4.2 Augmented Taylor Rules

An augmented Taylor rule departs from equation (40) in that:

$$\hat{R}_t = \phi_\pi \hat{\Pi}_t + \phi_y \left(\hat{Y}_t - \hat{A}_t\right) + \phi_c \left(\hat{C}_{rt} - \hat{C}_{kt}\right) + \phi_{LS} \left(\hat{LS}_t\right) \tag{41}$$

it potentially includes two new terms beyond those involving inflation and output gaps: the first is $\hat{C}_{rt} - \hat{C}_{kt}$, which measures changes in consumption inequality between Ricardians and Keynesians. The second is $\hat{LS}_t$, which measures fluctuations in the labor share.[43]

Figure 4: Welfare Under Augmented Taylor Rules




*Notes:* The figure shows how the part of welfare that is quadratic on gaps depends on the "augmented" Taylor rule parameters on consumption inequality, $\phi_c$, and on labor share, $\phi_{LS}$. Each line corresponds to the four Taylor rules evaluated in Table 2, columns (5)-(8), but in which we allow $\phi_c$ or $\phi_{LS}$ to be non-zero.

Figure 4 shows the welfare achieved by the four rules studied in Table 2, but additionally targeting either consumption inequality gap ($\phi_c \neq 0$) or the labor share ($\phi_{LS} \neq 0$).[44] When $\tau$ and $\gamma$ are not zero, it is possible to improve upon all the standard Taylor rules in Table 2 if:

- $\phi_c$ becomes negative as shown in the first panel in Figure 4. This is true across all four rules, with a maximum achieved around $\phi_c \in [-0.1, -0.05]$. The negative sign means that monetary policy should be set looser following a positive productivity shock since the shock raises consumption inequality.

[43]Note the augmented Taylor rule includes changes in the labor share and not the labor share gap for simplicity.

[44]In this section, we ignore $T_0$ when calculating welfare. Including $T_0$ would not change the qualitative results and in fact welfare gains are even larger when it is taken into account. The disadvantage, though, is that welfare gains become harder to interpret, because welfare losses can become negative as the inflation gap enters $T_0$ linearly.

However, there are limits to the gains from pushing $\phi_c$ into negative territory as evidenced by the hump in all four lines.

- $\phi_{LS}$ becomes positive as shown in the second panel in Figure 4. That means monetary policy should be set looser following a positive productivity shock since the shock lowers the labor share. Moreover, the larger the parameter on labor share the larger the gains (unless the parameter is set for extremely high values and even welfare losses beyond that level are small). Thus there is no similar hump shaped pattern in the welfare gains of targeting the labor share compared to targeting consumption inequality.

Hence, Figure 4 shows that targeting either the consumption inequality gap or the labor share is welfare increasing. The gains are large. For example, targeting the consumption inequality gap using $\phi_\pi = 1.5$, $\phi_y = 0.125$ and $\phi_c = -0.06$ generates a consumption equivalent gain of 2 x $10^{-3}$ percent and thereby closing the welfare loss of the same rule when $\phi_c = 0$ by about 79 percent, for a one percent productivity shock, approximately 1 standard deviation. Moreover, targeting the labor share gap seems at least as good as targeting consumption inequality —similarly computed relative gains are 82 percent for $\phi_{LS} = 3$. Gains relative to standard Taylor rules vary with the exact specification of standard rule considered, but they are always positive. Table 3 summarizes the gains of moving to either augmented Taylor rules.

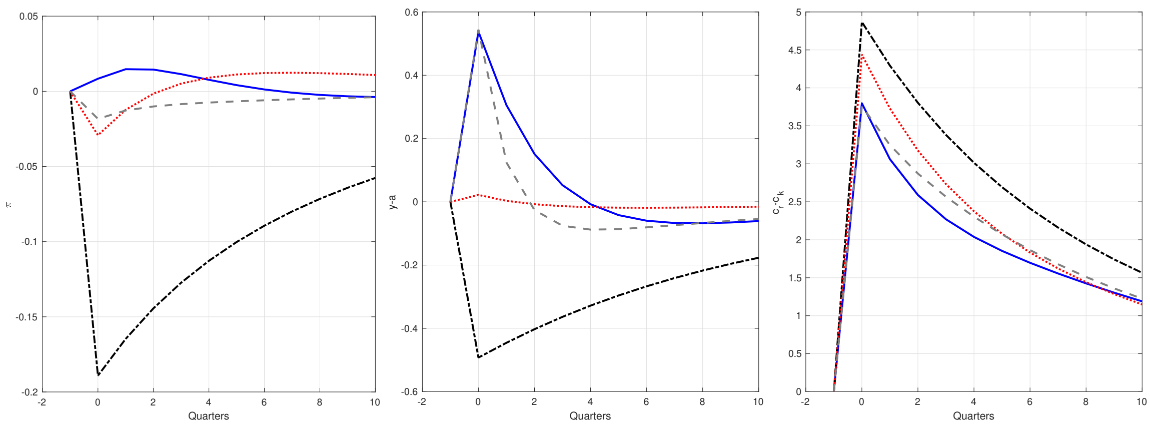
Table 3: Welfare in Taylor Rules - Augmented Rules

| | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| $\tau$ | 0 | 0 | 0 | 0.93 | 0.93 | 0.93 |
| $\gamma$ | 0 | 0 | 0 | 2.21 | 2.21 | 2.21 |
| $\phi_\pi$ | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 | 1.5 |
| $\phi_y$ | 0.125 | 0.125 | 0.125 | 0.125 | 0.125 | 0.125 |
| $\phi_c$ | 0 | $-0.06$ | 0 | 0 | $-0.06$ | 0 |
| $\phi_{LS}$ | 0 | 0 | 3 | 0 | 0 | 3 |
| Cons. Equiv. Loss | 0.1 | 0.1 | 0 | 1.5 | 0.2 | 0.2 |
| Cons. Equiv. $W - T_0$ | 0.1 | 0.1 | 0 | 0.3 | 0.1 | 0.1 |
| Inflation | 94.5 | 94.5 | 8.7 | 65.1 | 4.7 | 2.1 |
| Output | 5.5 | 5.5 | 91.3 | 4.5 | 0.1 | 7 |
| Inequality | 0 | 0 | 0 | 28.3 | 92.8 | 90.3 |
| Labor Share | 0 | 0 | 0 | 2.1 | 2.4 | 0.6 |
| Gains Relative to TR | 0 | 0 | 91.4 | 0 | 79.3 | 81.6 |

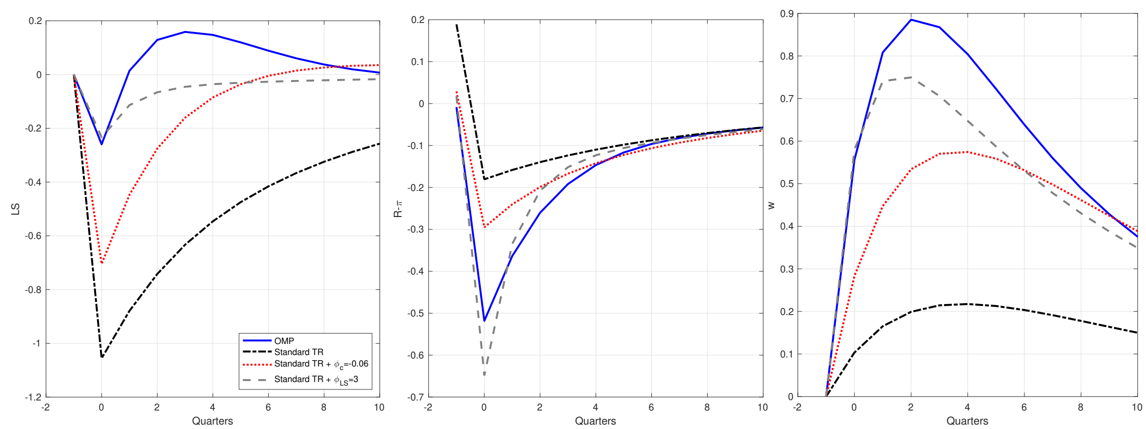
*Notes*: "Cons. Equiv. Loss" is the loss in yearly consumption multiplied by 10000 (e.g. 1 means 0.0001 percent of steady state consumption); "Cons. Equiv. $W - T_0$" is the dynamically relevant welfare component in the same units as the "Cons. Equiv. Loss"; "Inflation" is the share of welfare loss that is due to a non-zero inflation gap; "Output" is the share of welfare loss that is due to a non-zero output gap; "Inequality" is the share of welfare loss that is due to a non-zero inequality gap; and "Labor Share" is the share of welfare loss that is due to a non-zero labor share gap. "Gains Relative to TR" show the reduction in "Cons. Equiv. Loss" under each columns compared to column (1) for columns (1)-(3) and compared to column (4) for columns (4)-(6).

To understand better the benefits from targeting either consumption inequality or the labor share, Figure 5 compares impulse responses for one of the standard Taylor rules (using $\phi_\pi = 1.5$ and $\phi_y = 0.125$), two augmented Taylor rules—one targeting consumption inequality and another targeting the labor share— and finally those implied by the Optimal Monetary Policy explored in Section 3. For the augmented Taylor rules, the chosen parameter on the consumption inequality gap is $-0.06$ which is around the maximum welfare that can be achieved with targeting the consumption inequality gap, while the chosen parameter on the labor share is 3.

Figure 5: Impulse Response Functions: Standard Taylor rule, Augmented Taylor Rules and under Optimal Policy




(a) Inflation (b) Output Gap (c) Inequality




(d) Labor Share (e) Real Interest Rate (f) Wage Rate

*Notes:* The panels show impulse responses of, respectively, inflation, output gap, consumption inequality gap or "Inequality", labor share gap, the real interest rate and the wage rate. Four alternative policies are shown: the black dashed-dotted line corresponds to a standard Taylor rule using $\phi_\pi = 1.5$, $\phi_y = 0.125$ and $\phi_c = 0, \phi_{LS} = 0$, the red dotted line corresponds to an augmented Taylor Rule where $\phi_\pi = 1.5$, $\phi_y = 0.125$, $\phi_c = -0.06$, $\phi_{LS} = 0$, the grey dashed line corresponds to an augmented Taylor Rule where $\phi_\pi = 1.5$, $\phi_y = 0.125$, $\phi_c = 0$, $\phi_{LS} = 3$, and finally the solid blue line corresponds to Optimal Monetary Policy as defined in section 3.

Augmented Taylor rules raise welfare by making monetary policy more reactive to productivity shocks. Under the standard Taylor rule (black dashed line in Figure 5), a positive TFP shock leads to higher inequality and creates negative inflation, output and labor share gaps. By moving to a Taylor rule augmented with a negative parameter on the consumption inequality gap (red dashed line), real interest rates are set lower than they would be otherwise following the same positive TFP shock. This increases wages and stimulates demand, thereby closing inflation and output gaps. Higher wages also disproportionately help the poor and thus mitigate the increase in inequality. Interestingly, the augmented Taylor rule with an optimized parameter on inequality goes beyond closing the negative output gap and actually results in a small positive gap, similar to under optimal policy. Such a Taylor rule trades-off stabilizing macro gaps and stabilizing the consumption inequality gap.

Like the policy that targets consumption inequality, targeting the labor share (gray dashed line) leads to less volatility in all gaps, including in consumption inequality. Interestingly, targeting the labor share delivers much less volatile labor share and consumption inequality gaps than targeting consumption inequality directly, at the expense of a more volatile output gap. This dichotomy is generated by more volatile real interest rates and real wages under labor share targeting, although both deviate from steady state less persistently than under consumption inequality targeting. This greater persistence under consumption inequality targeting is a reflection of the inability of monetary policy to appropriately limit the volatility in consumption inequality.

Figure 5 helps explain the hump shape with respect to the parameter on targeting consumption inequality on the left panel of Figure 4. The hump is a product of two factors: first consumption inequality is quite persistent following a productivity shock and second monetary policy is not a very good tool at stemming consumption inequality deviations. The rise in consumption inequality following a positive productivity shock is very persistent because the productivity shock itself is persistent and pushes profits and wages above their steady state for long. A central bank that targets consumption inequality will loosen monetary policy also persistently and increasingly the more negative the parameter $\phi_c$. Persistent loose monetary policy has a small supportive effect on consumption inequality but drives up inflation and output gaps. Thus, beyond a certain point, the gains from having lower consumption inequality are completely swamped by the losses from macro destabilization. Targeting the labor share gap does not suffer from this problem because the labor share gap is not as persistent and monetary policy is quite effective in limiting the volatility of the labor share gap. Targeting the labor share instead of consumption inequality is hence also preferable because it is robust to the exact underlying deep parameters in the economy. The central bank can reliably target the labor share without knowing precisely the underlying extent of inequality and without having to worry that it is hurting excessively macro stability.

### 4.3 Full evaluation of augmented Taylor Rules

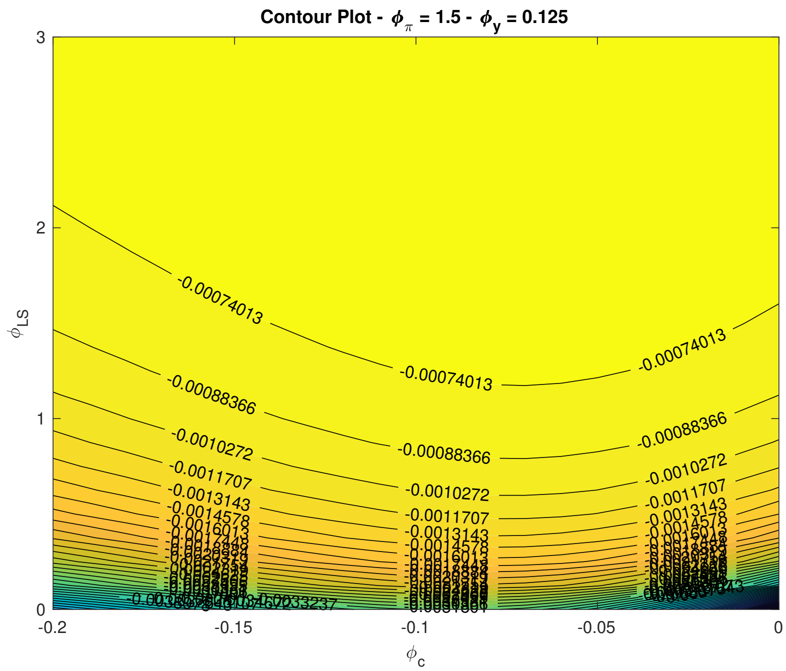
Taylor rules aim to stabilize key macroeconomic outcomes robustly in response to all shocks. Thus, finding that an inequality-augmented Taylor rule is superior under technology shocks only is not sufficient to argue that a central bank should include inequality in its targeting rule.

We augment the model described in Section 2 with (i) a cost push and (ii) a demand shock, and conduct a standard ex-ante evaluation of Taylor rules.[45] Figure 6 shows how welfare varies in such an econ-

[45]We follow Galí, 2015. See Appendix C for details.

omy when varying the parameters in the augmented Taylor rule for consumption inequality and labor share. As the central bank begins targeting consumption inequality (moving from right to left in the figure), welfare rises initially and then falls, much as when considering TFP shocks alone in Figure 4. However, unlike when there are TFP shocks alone, targeting the labor share is clearly preferable under the full set of shocks, since higher welfare can always be reached by raising the parameter on labor share, even without targeting consumption inequality.[46]

Figure 6: Welfare Loss and Policy Reaction Parameters



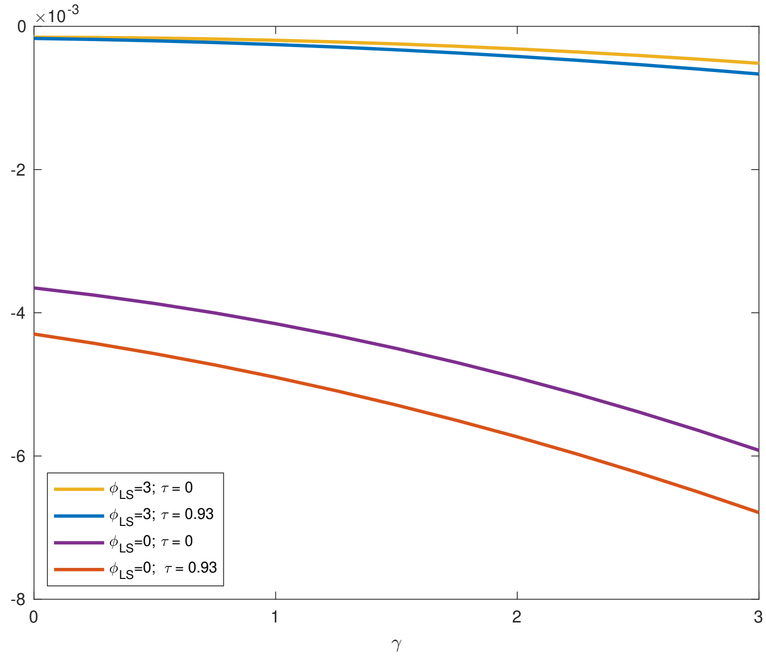

*Notes:* The figure reports a contour plot for the consumption-equivalent welfare loss as a function of the reaction terms to consumption inequality, $\phi_c$, and labor share gap, $\phi_{LS}$, as described by equation (41).

Targeting the labor share dampens the losses from the underlying drivers of inequality and is robust to the degree of inequality. Figure 7 shows how ex-ante welfare losses vary with $\gamma$ and $\tau$ for the standard and labor-share-augmented Taylor rules. The purple line shows the welfare loss from using a standard Taylor rule depending on the degree of skill bias in wages ($\gamma$) when there is no steady-state inequality ($\tau = 0$). Under that case, welfare losses rise by about 50% when moving from no skill-bias in wages ($\gamma = 0$) to our calibrated degree of skill-bias in wages. The same holds for the red line where, instead of no steady-state inequality, we evaluate losses under our calibrated degree of steady-state inequality ($\tau = 0.93$). Targeting the labor share, on the other hand, dampens the impact on welfare of both higher degree of skill-bias in wages and larger degree of steady-state inequality, as seen by the yellow and blue lines which are close together (effect of $\tau$) and much flatter (effect of $\gamma$). Therefore, targeting the labor share improves both the welfare for any given underlying drivers of inequality and it also makes welfare robust to different

[46] Appendix Figure B2 shows that the hump in the consumption inequality parameter is indeed a characteristic inherent to tech shocks, while for labor share targeting a higher parameter is always superior under all three shocks, which thus means that labor share targeting becomes much preferred when including demand and cost push shocks in addition to technology shocks.

degrees of such drivers.

Figure 7: Welfare Loss as function of Inequality Parameters across Taylor Rules




*Notes:* The figure reports the welfare loss as a function of the skill bias parameter, $\gamma$, under four parameterizations. The blue and red lines correspond to economies with the calibrated values for $\tau$ where the central bank reacts (or not) to the labor share, as described by equation (41). The yellow and purple lines correspond to economies with no steady state inequality $\tau = 0$ where the central bank reacts (or not) to the labor share, as described by equation (41).

## 5 Conclusion

Should inequality factor into central banks' decisions? We analyzed this question using a stylized Two-Agent New Keynesian Model. We studied both (i) optimal monetary policy whereby the central bank cares equally about all agents in the economy and (ii) Taylor rules whereby the central bank sets the interest rate based on the state of the aggregate economy.

We find that there are economically small aggregate welfare gains in targeting consumption inequality if the central bank already implements optimal monetary policy ignoring inequality. A central bank should place a non-zero optimal social weight on the consumption inequality gap. However, the welfare gain of taking inequality explicitly into account is only about 1.2 percent of the loss under optimal monetary policy targeted to the average agent.

On the other hand, if the central bank implements monetary policy through a standard Taylor rule, then augmenting it with either an inequality target or a labor share target can lead to higher welfare. Beyond targeting inflation and output gaps, the central bank can achieve higher welfare if it places a small negative weight on consumption inequality. This means that following a positive TFP shock that increases consumption inequality, the central bank should reduce the policy rate. This would reduce the welfare loss by about 79 percent compared to that of a standard Taylor rule. Targeting the labor share seems at least as good as targeting consumption inequality, delivering about 82 percent of similarly computed relative gains.

Targeting the labor share is though superior ex-ante when considering a fuller constellation of business cycle shocks. Under demand, cost push and technology shocks, labor share targeting achieves higher welfare and is also much more robust to the exact magnitude of inequality in the economy. Interestingly, our results do not depend on the existence of unemployment and how monetary policy can mitigate its volatility, which could be another motive to make monetary policy inequality-sensitive. Our results also speak to the usefulness of dual mandates with a particular emphasis on labor market outcomes, such as the U.S. Federal Reserve's new framework.

These conclusions are of course model specific. Although we believe the model captures important dimensions of inequality, its appeal is that it is simple enough to study optimal policy. It will be important for future research to assess whether these findings can be generalized. First, we modeled steady-state inequality arising from an unequal distribution of equity holdings and hence profits in an economy with a simplistic wealth distribution and no aggregate savings. This could be extended to inequality from other income sources, in particular from differentiated labor, and a richer form of wealth inequality, capturing differentiated savings and also holdings of illiquid assets like housing. The movements in other asset prices in response to monetary policy could also be modelled beyond stock prices. Second, we restricted our analysis to monetary policy. It would be important to study how the conclusions above change when monetary policy is used in coordination with other policy tools.

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# A Details in Deriving Optimal and RANK-Optimal Policies

## A.1 Approach

In this section we approximate the welfare of a utilitarian central planner. There are several steps in this derivation. First, we consider the overall utility within a given period and obtain an expression that involves a linear term in output gap $\hat{Y}_t - \hat{A}_t$. Second, we take a second order approximation to the New Keynesian Phillips curve that features a linear term in output gap. Finally, we substitute the New Keynesian Phillips curve into the discounted sum of the expression obtained in the first step.

We will often sum all terms independent of monetary policy in the term t.i.p.. We will also use the sign $\approx$ when we take Taylor approximations or ignore terms of order higher than 2 and when we discard t.i.p. terms. Also recall that we define $\hat{x}_t \equiv \ln x_t - \ln \overline{x}$ and that $\frac{x_t - \overline{x}}{\overline{x}} \approx \hat{x}_t + \frac{1}{2}\hat{x}_t^2$. In order to obtain a welfare function we follow these steps. First, as in the literature we take a second order approximation of the average agent's lifetime utility. Second, because of steady state inequality, we will have a linear term. that will be substituted using the methodology of Benigno and Woodford, 2005.

## A.2 Second order approximation

For notational convenience we first ignore the time dimension and we add and subtract the steady state consumption utilities.

$$W_t = \lambda \ln C_{kt} + (1-\lambda)\ln C_{rt} - \chi \frac{N_t^{1+\phi}}{1+\phi} = \lambda \hat{C}_{kt} + (1-\lambda)\hat{C}_{rt} - \chi \frac{N_t^{1+\phi}}{1+\phi} + t.i.p. \tag{42}$$

We then take a second order approximation of the labor dis-utility part (the consumption part is just itself). And we use the fact the in steady state $\chi N^{1+\phi} = 1-\alpha$.

$$\begin{aligned} W_t &\approx \lambda \hat{C}_{kt} + (1-\lambda)\hat{C}_{rt} - \chi N^{\phi}(N_t - N) - \frac{1}{2}\chi\phi N^{\phi-1}(N_t - N)^2 \\ &\approx \lambda \hat{C}_{kt} + (1-\lambda)\hat{C}_{rt} - \chi N^{1+\phi}\hat{N}_t - \frac{1+\phi}{2}\chi N^{\phi+1}\hat{N}_t^2 \approx \lambda \hat{C}_{kt} + (1-\lambda)\hat{C}_{rt} - (1-\alpha)\hat{N}_t - \frac{1+\phi}{2}(1-\alpha)\hat{N}_t^2 \end{aligned}$$

Now we use that $(1-\alpha)\hat{N} = \hat{Y} - \hat{A}$.

$$W_t \approx \lambda \hat{C}_{kt} + (1-\lambda)\hat{C}_{rt} - \hat{Y}_t - \frac{1+\phi}{2(1-\alpha)}(\hat{Y}_t - \hat{A}_t)^2 + t.i.p. \tag{43}$$

Let us now take a second order approximation to the aggregate resource constraint.

$$\begin{aligned} Y_t\left[1 - \frac{\psi_p}{2}(\Pi_t - 1)^2\right] &= \lambda C_{kt} + (1-\lambda)C_{rt} \\ Y_t - \overline{Y} - \frac{\psi_p}{2}\overline{Y}(\Pi_t - \Pi)^2 &= \lambda(C_{kt} - \overline{C}_k) + (1-\lambda)(C_{rt} - \overline{C}_r) \\ \hat{Y}_t + \frac{1}{2}\hat{Y}_t^2 - \frac{\psi_p}{2}\hat{\Pi}_t^2 &= \frac{\lambda \overline{C}_k}{\overline{Y}}(\hat{C}_{kt} + \frac{1}{2}\hat{C}_{kt}^2) + \frac{(1-\lambda)\overline{C}_r}{\overline{Y}}(\hat{C}_{rt} + \frac{1}{2}\hat{C}_{rt}^2) \end{aligned} \tag{44}$$

Notice that $\frac{\overline{C}_k}{\overline{Y}} = \frac{\overline{Y}-\overline{Y}+\overline{w}\overline{N}-T_p\overline{Y}+(1-\tau)\overline{d}}{\overline{Y}} = \frac{\overline{Y}-\overline{d}+(1-\tau)\overline{d}}{\overline{Y}} = 1-\tau\frac{\overline{d}}{\overline{Y}}$. By squaring the same equation and substituting in itself we obtain the following.

$$\begin{aligned}
\hat{Y}_t^2 &= \left(\lambda\frac{\overline{C}_k}{\overline{Y}}\right)^2 \hat{C}_{kt}^2 + \left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)^2 \hat{C}_{rt}^2 + 2\left(\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\hat{C}_{kt}\hat{C}_{rt} \\
\hat{Y}_t &= \left(\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\hat{C}_{kt} + \left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\hat{C}_{rt} + \frac{\psi_p}{2}\hat{\Pi}_t^2 + \left(\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{1}{2}(\hat{C}_{rt}-\hat{C}_{kt})^2 \\
\hat{Y}_t &= \lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\hat{C}_{kt} + \left(1-\lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\right)\hat{C}_{rt} + \frac{\psi_p}{2}\hat{\Pi}_t^2 + \lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\left(1-\lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\right)\frac{1}{2}(\hat{C}_{rt}-\hat{C}_{kt})^2
\end{aligned} \tag{45}$$

Now we substitute the above simplification into the welfare.

$$W_t \approx -\lambda\tau\frac{\overline{d}}{\overline{Y}}(\hat{C}_{rt}-\hat{C}_{kt}) - \frac{\psi_p}{2}\hat{\Pi}_t^2 - \lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\left(1-\lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\right)\frac{1}{2}(\hat{C}_{rt}-\hat{C}_{kt})^2 - \frac{1+\phi}{2(1-\alpha)}(\hat{Y}_t-\hat{A}_t)^2 + t.i.p. \tag{46}$$

We now want to substitute the first term. We rearrange the resource constraint.

$$\begin{aligned}
\frac{C_{rt}}{C_{kt}} &= \frac{-\lambda}{1-\lambda} + \frac{1}{1-\lambda}\frac{Y_t}{C_{kt}}\left(1-\frac{\psi_p}{2}(\Pi_t-1)^2\right) \\
\exp\ln\left(\frac{C_{rt}}{C_{kt}}\right) &= \frac{-\lambda}{1-\lambda} + \frac{1}{1-\lambda}\exp\ln\left(\frac{Y_t}{C_{kt}}\right)\left(1-\frac{\psi_p}{2}(\Pi_t-1)^2\right) \\
\exp\ln\left(\frac{C_{rt}}{C_{kt}}\right)\exp-\ln\left(\frac{\overline{C}_r}{\overline{C}_k}\right) &= \frac{-\lambda}{1-\lambda}\exp-\ln\left(\frac{\overline{C}_r}{\overline{C}_k}\right) + \\
&\quad + \frac{1}{1-\lambda}\exp\ln\left(\frac{Y_t}{C_{kt}}\right)\left(1-\frac{\psi_p}{2}(\Pi_t-1)^2\right)\exp-\ln\left(\frac{\overline{C}_r}{\overline{C}_k}\right)\exp\ln\left(\frac{\overline{Y}}{\overline{C}_k}\right)\exp-\ln \\
\exp\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &= \frac{-\lambda}{1-\lambda}\frac{\overline{C}_k}{\overline{C}_r} + \frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\exp\left(\hat{Y}_t-\hat{C}_{kt}\right)\left(1-\frac{\psi_p}{2}(\Pi_t-1)^2\right) \\
\left(\hat{C}_{rt}-\hat{C}_{kt}\right) + \frac{1}{2}\left(\hat{C}_{rt}-\hat{C}_{kt}\right)^2 &\approx \frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left[\left(\hat{Y}_t-\hat{C}_{kt}\right) + \frac{1}{2}\left(\hat{Y}_t-\hat{C}_{kt}\right)^2 - \frac{\psi_p}{2}\hat{\Pi}_t^2\right] \qquad (47) \\
\left(\hat{C}_{rt}-\hat{C}_{kt}\right) + \frac{1}{2}\left(\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)^2\left(\hat{Y}_t-\hat{C}_{kt}\right)^2 &\approx \frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left[\left(\hat{Y}_t-\hat{C}_{kt}\right) + \frac{1}{2}\left(\hat{Y}_t-\hat{C}_{kt}\right)^2 - \frac{\psi_p}{2}\hat{\Pi}_t^2\right] \\
\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &\approx \frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left[-\left(\hat{C}_{kt}-\hat{Y}_t\right) + \frac{1}{2}\left(\hat{C}_{kt}-\hat{Y}_t\right)^2 - \frac{\psi_p}{2}\hat{\Pi}_t^2\right] - \frac{1}{2}\left(\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)^2\left(\hat{C}_{kt} - \right.
\end{aligned}$$

(48)

Now consider the budget constraint of the Keynesian agent. A second order approximation gives the following.

$$
\begin{aligned}
\left(\hat{C}_{kt}-\hat{Y}_t\right) &= \hat{A}_t + \frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\hat{LS}_t + \\
&+\left\{\frac{1}{2}\gamma^2\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left(1-\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\right)\right\}\hat{A}_t^2+\left\{\frac{1}{2}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\left[1-\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]\right\}\hat{LS}_t^2+ \\
&-\frac{1}{2}\frac{(1-\tau)\,\psi_p}{\frac{\overline{C}_k}{\overline{Y}}}\hat{\Pi}_t^2-\gamma\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left[1-\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]\hat{A}_t\hat{LS}_t \\
\left(\hat{C}_{kt}-\hat{Y}_t\right)^2 &= \left[\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\right]^2\hat{LS}_t^2-2\gamma\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\hat{A}_t\hat{LS}_t+\gamma^2\frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2}\hat{A}_t^2
\end{aligned}
$$

We are now ready to make the substitution into the inequality equation from the previous steps.

$$
\begin{aligned}
\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &\approx \frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left[-\left(\hat{C}_{kt}-\hat{Y}_t\right)+\frac{1}{2}\left(\hat{C}_{kt}-\hat{Y}_t\right)^2-\frac{\psi_p}{2}\hat{\Pi}_t^2\right]-\frac{1}{2}\left(\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)^2\left(\hat{C}_{kt}-\hat{Y}_t\right)^2 \\
\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &\approx -\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left(\hat{C}_{kt}-\hat{Y}_t\right)+\frac{1}{2}\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\left(1-\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)\left(\hat{C}_{kt}-\hat{Y}_t\right)^2-\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\frac{\psi_p}{2}\hat{\Pi}_t^2 \\
\left(\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)^{-1}\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &\approx -\left(\hat{C}_{kt}-\hat{Y}_t\right)+\frac{1}{2}\left(1-\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)\left(\hat{C}_{kt}-\hat{Y}_t\right)^2-\frac{\psi_p}{2}\hat{\Pi}_t^2 \\
\left(\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)^{-1}\left(\hat{C}_{rt}-\hat{C}_{kt}\right) &\approx -\hat{A}_t-\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\hat{LS}_t+ \\
&-\left\{\frac{1}{2}\gamma^2\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left(1-\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\right)\right\}\hat{A}_t^2-\left\{\frac{1}{2}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\left[1-\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]\right\}\hat{LS}_t^2+ \\
&+\frac{1}{2}\frac{(1-\tau)\,\psi_p}{\frac{\overline{C}_k}{\overline{Y}}}\hat{\Pi}_t^2-\gamma\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left[1-\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]\hat{A}_t\hat{LS}_t \\
&+\frac{1}{2}\left(1-\frac{1}{1-\lambda}\frac{\overline{Y}}{\overline{C}_r}\right)\left(\left[\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\right]^2\hat{LS}_t^2-2\gamma\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\hat{A}_t\hat{LS}_t+\gamma^2\frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2}\hat{A}_t^2\right)^2-\frac{\psi_p}{2}\hat{\Pi}_t^2
\end{aligned}
\tag{49}
$$

We now collect and rearranges the terms.

$$\begin{aligned}
\hat{C}_{rt} - \hat{C}_{kt} = & -\frac{(1-\alpha)\,\tau}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}}\hat{LS}_t + \\
& -\frac{1}{2}\left[\frac{1}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} - \frac{1-2\lambda\frac{\overline{C}_k}{\overline{Y}}}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)^2}\frac{\tau^2\,(1-\alpha)^2}{\frac{\overline{C}_k}{\overline{Y}}^2}\right]\hat{LS}_t^2 + \\
& + \left[\frac{\gamma}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left[1 - \frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right] + \frac{\tilde{\lambda}}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)^2}\gamma\frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2}\tau\right]\hat{A}_t\hat{LS}_t + \\
& -\frac{1}{2}\frac{\psi_p}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\left(1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}\right)\hat{\Pi}_t^2
\end{aligned} \tag{50}$$

Substituting this expression into the welfare function we obtain

$$\begin{aligned}
W_t \approx & -\lambda\tau\frac{\overline{d}}{\overline{Y}}(\hat{C}_{rt} - \hat{C}_{kt}) - \frac{\psi_p}{2}\hat{\Pi}_t^2 - \lambda\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)\left(1 - \lambda\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)\right)\frac{1}{2}(\hat{C}_{rt} - \hat{C}_{kt})^2 - \frac{1+\phi}{2\,(1-\alpha)}(\hat{Y}_t - \hat{A}_t)^2 + t.i.p. \\
W_t \approx & \lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{(1-\alpha)\,\tau}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}}\hat{LS}_t + \\
& + \frac{1}{2}\lambda\tau\frac{\overline{d}}{\overline{Y}}\left[\frac{1}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} - \frac{1-2\lambda\frac{\overline{C}_k}{\overline{Y}}}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)^2}\frac{\tau^2\,(1-\alpha)^2}{\frac{\overline{C}_k}{\overline{Y}}^2}\right]\hat{LS}_t^2 + \\
& - \lambda\tau\frac{\overline{d}}{\overline{Y}}\left[\frac{\gamma}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}}\left[1 - \frac{\tau\,(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right] + \frac{\tilde{\lambda}}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)^2}\gamma\frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2}\tau\right]\hat{A}_t\hat{LS}_t + \\
& + \lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{1}{2}\frac{\psi_p}{\left((1-\lambda)\,\frac{\overline{C}_r}{\overline{Y}}\right)}\left(1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}\right)\hat{\Pi}_t^2 \\
& - \frac{\psi_p}{2}\hat{\Pi}_t^2 - \lambda\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)\left(1 - \lambda\left(1 - \tau\frac{\overline{d}}{\overline{Y}}\right)\right)\frac{1}{2}(\hat{C}_{rt} - \hat{C}_{kt})^2 - \frac{1+\phi}{2\,(1-\alpha)}(\hat{Y}_t - \hat{A}_t)^2 + t.i.p.
\end{aligned} \tag{51}$$

Now we collect terms and take the time summation in.

$$
\begin{aligned}
W = & \sum_0^\infty \beta^t \left\{ -\frac{\psi_p}{2} + \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \frac{\psi_p}{(1-\lambda)\frac{\overline{C}_r}{\overline{Y}}} (1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}) \right\} \hat{\Pi}_t^2 \\
& + \sum_0^\infty \beta^t \left\{ -\frac{1+\phi}{2(1-\alpha)} \right\} (\hat{Y}_t - \hat{A}_t)^2 + \sum_0^\infty \beta^t \left\{ -\lambda \frac{\overline{C}_k}{\overline{Y}} ((1-\lambda)\frac{\overline{C}_r}{\overline{Y}}) \frac{1}{2} \right\} (\hat{C}_{rt} - \hat{C}_{kt})^2 \\
& + \sum_0^\infty \beta^t \left\{ +\lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \left[ \frac{1}{(1-\lambda)\frac{\overline{C}_r}{\overline{Y}}} \frac{\tau(1-\alpha)}{\overline{\tilde{C}}_k} - \frac{1 - 2\lambda \frac{\overline{C}_k}{\overline{Y}}}{\left((1-\lambda)\frac{\overline{C}_r}{\overline{Y}}\right)^2} \frac{\tau^2 (1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \right] \right\} \hat{LS}_t^2 \\
& + \sum_0^\infty \beta^t \left\{ -\lambda\tau \frac{\overline{d}}{\overline{Y}} \left[ \frac{\gamma}{(1-\lambda)\frac{\overline{C}_r}{\overline{Y}}} \frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}} \left[ 1 - \frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} \right] + \frac{\lambda \frac{\overline{C}_k}{\overline{Y}}}{\left((1-\lambda)\frac{\overline{C}_r}{\overline{Y}}\right)^2} \gamma \frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \tau \right] \right\} \hat{A}_t \hat{LS}_t \\
& + \sum_0^\infty \beta^t \left\{ +\lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{(1-\alpha)\tau}{\left((1-\lambda)\frac{\overline{C}_r}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}} \right\} \hat{LS}_t
\end{aligned}
$$

## A.3 Getting rid of the linear term

We now get rid of the linear term in the following way. 1) We take a second order approximation of the Phillips curve, which will have a linear term in the inequality term; 2) we substitute that to the welfare function. Let us start by the Phillips curve.

$$
\begin{aligned}
\Pi_t [\Pi_t - 1] &= \beta \left(\frac{C_{rt+1}}{C_{rt}}\right)^{-1} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} [\Pi_{t+1} - 1] + \frac{\theta_p}{\psi_p} \left[ \frac{1}{1-\alpha} \frac{w_t}{A_t^{\frac{1}{1-\alpha}}} Y_t^{\frac{\alpha}{1-\alpha}} - (1+T_p) \frac{\theta_p - 1}{\theta_p} \right] \\
\Pi_t [\Pi_t - 1] &= \beta \left(\frac{C_{rt+1}}{C_{rt}}\right)^{-1} \frac{Y_{t+1}}{Y_t} \Pi_{t+1} [\Pi_{t+1} - 1] + \frac{\theta_p}{\psi_p} \left[ \frac{1}{1-\alpha} \frac{w_t N_t}{Y_t} - (1+T_p) \frac{\theta_p - 1}{\theta_p} \right] \\
\Pi_t [\Pi_t - 1] \frac{C_{rt}^{-1} Y_t}{C_r^{-1} Y} &= \beta \frac{C_{rt+1}^{-1} Y_{t+1}}{C_r^{-1} Y} \Pi_{t+1} [\Pi_{t+1} - 1] + \frac{C_{rt}^{-1} Y_t}{C_r^{-1} Y} \frac{\theta_p}{\psi_p} \left[ \frac{1}{1-\alpha} LS_t - (1+T_p) \frac{\theta_p - 1}{\theta_p} \right] && (52) \\
T_t &= \beta T_{t+1} + TT_t = \sum_0^\infty \beta^i TT_{t+i} && (53)
\end{aligned}
$$

Let's consider the term $T_t$. A second order approximation yields

$$
\Pi_t [\Pi_t - 1] \frac{C_{rt}^{-1} Y_t}{C_r^{-1} Y} \approx \hat{\Pi}_t + \frac{3}{2} \hat{\Pi}_t^2 + \hat{\Pi}_t \hat{Y}_t - \hat{\Pi}_t \hat{C}_{rt} \quad (54)
$$

Let's now consider the term $TT_t$. A second order approximation yields

$$
\frac{C_{rt}^{-1} Y_t}{C_r^{-1} Y} \frac{\theta_p}{\psi_p} \left[ \frac{1}{1-\alpha} LS_t - (1+T_p) \frac{\theta_p - 1}{\theta_p} \right] \approx \frac{\theta_p}{\psi_p} \left( \hat{LS}_t + \frac{1}{2} \hat{LS}_t^2 + \hat{LS}_t (\hat{Y}_t - \hat{C}_{rt}) \right) \quad (55)
$$

By combining the two expressions, and making use of previously derived conditions, we have

$$\frac{\psi_p}{\theta_p}\tilde{T}_0 = \sum_{t=0}^{\infty}\beta^t\left\{\hat{LS}_t + \left(\frac{1}{2} + \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\right)\hat{LS}_t^2 - \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\gamma\frac{(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\hat{A}_t\hat{LS}_t\right\}$$

$$\sum_{t=0}^{\infty}\beta^t\hat{LS}_t = \frac{\psi_p}{\theta_p}\tilde{T}_0 + \sum_{t=0}^{\infty}\beta^t\left\{-\left(\frac{1}{2} + \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\right)\hat{LS}_t^2 + \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\gamma\frac{(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\hat{A}_t\hat{LS}_t\right\}$$

$$\begin{aligned}
&\sum_{t=0}^{\infty}\beta^t\left\{\lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{(1-\alpha)\,\tau}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}}\right\}\hat{LS}_t \\
&= \left\{\lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{(1-\alpha)\,\tau}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}}\right\}\frac{\psi_p}{\theta_p}\tilde{T}_0 + \\
&+ \left\{\lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{(1-\alpha)\,\tau}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}}\right\}\sum_{t=0}^{\infty}\beta^t\left\{-\left(\frac{1}{2} + \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\frac{(1-\alpha)\,\tau}{\frac{\overline{C}_k}{\overline{Y}}}\right)\hat{LS}_t^2 + \frac{\lambda\frac{\overline{C}_k}{\overline{Y}}}{1-\lambda\frac{\overline{C}_k}{\overline{Y}}}\gamma\frac{(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\hat{A}_t\hat{LS}_t\right\}
\end{aligned}$$

Finally, we now substitute the linear term into the welfare function.

$$
\begin{aligned}
W = & \sum_{0}^{\infty} \beta^t \left\{ -\frac{\psi_p}{2} + \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \frac{\psi_p}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} (1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}) \right\} \hat{\Pi}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1+\phi}{2(1-\alpha)} \right\} (\hat{Y}_t - \hat{A}_t)^2 + \sum_{0}^{\infty} \beta^t \left\{ -\lambda \frac{\overline{C}_k}{\overline{Y}} (1 - \lambda \frac{\overline{C}_k}{\overline{Y}}) \frac{1}{2} \right\} (\hat{C}_{rt} - \hat{C}_{kt})^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ +\lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \left[ \frac{1}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} - \frac{1 - 2\lambda \frac{\overline{C}_k}{\overline{Y}}}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right)^2} \frac{\tau^2 (1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \right] \right\} \hat{LS}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\lambda\tau \frac{\overline{d}}{\overline{Y}} \left[ \frac{\gamma}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}} \left[ 1 - \frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} \right] + \frac{\lambda \frac{\overline{C}_k}{\overline{Y}}}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right)^2} \gamma \frac{(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \tau \right] \right\} \hat{A}_t \hat{LS}_t + \\
& + \left\{ \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{(1-\alpha)\tau}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right) \frac{\overline{C}_k}{\overline{Y}}} \right\} \sum_{t=0}^{\infty} \beta^t \left\{ -\left( \frac{1}{2} + \frac{\lambda \frac{\overline{C}_k}{\overline{Y}}}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \frac{(1-\alpha)\tau}{\frac{\overline{C}_k}{\overline{Y}}} \right) \hat{LS}_t^2 + \frac{\lambda \frac{\overline{C}_k}{\overline{Y}}}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \gamma \frac{(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} \hat{A}_t \hat{LS}_t \right\} \\
& + \left\{ \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{(1-\alpha)\tau}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right) \frac{\overline{C}_k}{\overline{Y}}} \right\} \frac{\psi_p}{\theta_p} \tilde{T}_0 \\
= & \sum_{0}^{\infty} \beta^t \left\{ -\frac{\psi_p}{2} + \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \frac{\psi_p}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} (1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}) \right\} \hat{\Pi}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1+\phi}{2(1-\alpha)} \right\} (\hat{Y}_t - \hat{A}_t)^2 + \sum_{0}^{\infty} \beta^t \left\{ -\lambda \frac{\overline{C}_k}{\overline{Y}} (1 - \lambda \frac{\overline{C}_k}{\overline{Y}}) \frac{1}{2} \right\} (\hat{C}_{rt} - \hat{C}_{kt})^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1}{2} \frac{\lambda\tau \frac{\overline{d}}{\overline{Y}}}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right)^2} \frac{\tau^2 (1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \right\} \hat{LS}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{\gamma}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}} \left[ 1 - \frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} \right] \right\} \hat{A}_t \hat{LS}_t + \\
& + \left\{ \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{(1-\alpha)\tau}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right) \frac{\overline{C}_k}{\overline{Y}}} \right\} \frac{\psi_p}{\theta_p} \tilde{T}_0 \\
= & \sum_{0}^{\infty} \beta^t \left\{ -\frac{\psi_p}{2} + \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{1}{2} \frac{\psi_p}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} (1 - \frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}) \right\} \hat{\Pi}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1+\phi}{2(1-\alpha)} \right\} (\hat{Y}_t - \hat{A}_t)^2 + \sum_{0}^{\infty} \beta^t \left\{ -\lambda \frac{\overline{C}_k}{\overline{Y}} (1 - \lambda \frac{\overline{C}_k}{\overline{Y}}) \frac{1}{2} \right\} (\hat{C}_{rt} - \hat{C}_{kt})^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1}{2} \frac{\lambda\tau \frac{\overline{d}}{\overline{Y}}}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right)^2} \frac{\tau^2 (1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \left[ \hat{LS}_t + \frac{\lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{\gamma}{1-\lambda \frac{\overline{C}_k}{\overline{Y}}} \frac{1-\alpha}{\frac{\overline{C}_k}{\overline{Y}}} \left[ 1 - \frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}} \right]}{\frac{\lambda\tau \frac{\overline{d}}{\overline{Y}}}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right)^2} \frac{\tau^2(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2}} \hat{A}_t \right]^2 \right\} \\
& + \left\{ \lambda\tau \frac{\overline{d}}{\overline{Y}} \frac{(1-\alpha)\tau}{\left(1-\lambda \frac{\overline{C}_k}{\overline{Y}}\right) \frac{\overline{C}_k}{\overline{Y}}} \right\} \frac{\psi_p}{\theta_p} \tilde{T}_0
\end{aligned}
$$

Finally we can simplify the coefficient in front of the tech shock and write the welfare function as:

$$
\begin{aligned}
W = & \sum_{0}^{\infty} \beta^t \left\{ -\frac{\psi_p}{2} + \lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{1}{2}\frac{\psi_p}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)}(1-\frac{(1-\tau)}{\frac{\overline{C}_k}{\overline{Y}}}) \right\} \hat{\Pi}_t^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1+\phi}{2\,(1-\alpha)} \right\} (\hat{Y}_t - \hat{A}_t)^2 + \sum_{0}^{\infty} \beta^t \left\{ -\lambda\frac{\overline{C}_k}{\overline{Y}}(\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right))\frac{1}{2} \right\} (\hat{C}_{rt} - \hat{C}_{kt})^2 \\
& + \sum_{0}^{\infty} \beta^t \left\{ -\frac{1}{2}\frac{\lambda\tau\frac{\overline{d}}{\overline{Y}}}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)^2}\frac{\tau^2\,(1-\alpha)^2}{\left(\frac{\overline{C}_k}{\overline{Y}}\right)^2} \left[ \hat{LS}_t + \frac{\gamma\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\frac{\overline{C}_k}{\overline{Y}}}{1-\alpha}\left[1-\frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]}{\tau^2}\hat{A}_t \right]^2 \right\} \\
& + \left\{ \lambda\tau\frac{\overline{d}}{\overline{Y}}\frac{(1-\alpha)\,\tau}{\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\overline{C}_k}{\overline{Y}}} \right\} \frac{\psi_p}{\theta_p}\tilde{T}_0
\end{aligned}
$$

## A.4 Welfare Weights

From above, the welfare function can be re-written as:

$$
W \approx -\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{W_\Pi\hat{\Pi}_t^2 + W_Y(\hat{Y}_t-\hat{A}_t)^2 + W_{\Delta_c}(\hat{C}_{rt}-\hat{C}_{kt})^2 + W_{LS}(\widehat{LS}_t - \widehat{LS}_t^*)^2\right\} + T_0 + t.i.p.
$$

where:

$$
W_\Pi = \psi_p - \psi_p\Psi(\tau)\left(1-\frac{\overline{d}}{\overline{Y}}\right) \tag{56}
$$

$$
W_Y = \frac{1+\phi}{1-\alpha} \tag{57}
$$

$$
W_{\Delta_c} = \lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\left(1-\lambda\left(1-\tau\frac{\overline{d}}{\overline{Y}}\right)\right) \tag{58}
$$

$$
W_{LS} = (1-\alpha)^2\,\frac{\Psi(\tau)^2}{\lambda\tau\frac{\overline{d}}{\overline{Y}}} \tag{59}
$$

$$
\Psi(\tau) \equiv \frac{\lambda\tau\frac{\overline{d}}{\overline{Y}}}{1-\lambda+\lambda\tau\frac{\overline{d}}{\overline{Y}}}\left(\frac{\tau}{1-\tau\frac{\overline{d}}{\overline{Y}}}\right) \tag{60}
$$

Finally, the welfare relevant labor share is: $\widehat{LS}_t^* = \frac{\gamma\left(1-\lambda\frac{\overline{C}_k}{\overline{Y}}\right)\frac{\frac{\overline{C}_k}{\overline{Y}}}{1-\alpha}\left[1-\frac{\tau(1-\alpha)}{\frac{\overline{C}_k}{\overline{Y}}}\right]}{\tau^2}\hat{A}_t$.

## A.5 Maximization Problem

The Lagrangian of the maximization problem can be written as follows.

$$L = \sum_0^\infty \beta^t \left\{ -\frac{1}{2} W_\Pi \hat{\Pi}_t^2 - \frac{1}{2} W_Y \left( \hat{Y}_t - \hat{A}_t \right)^2 - \frac{1}{2} W_{\Delta_c} (\hat{C}_{rt} - \hat{C}_{kt})^2 - \frac{1}{2} W_{LS} (\hat{LS}_t - \hat{LS}_t^*)^2 \right\} + \tag{61}$$

$$+ \left\{ \lambda \tau \frac{\overline{d}}{\overline{\overline{Y}}} \frac{(1-\alpha)\tau}{\left(1 - \lambda + \lambda \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right)\left(1 - \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right)} \right\} \frac{\psi_p}{\theta_p} \tilde{T}_0 + \tag{62}$$

$$+ \sum_0^\infty \beta^t \mu_t^1 \left\{ \hat{\Pi}_t - \beta \hat{\Pi}_{t+1} - \frac{\theta_p}{\psi_p} \hat{LS}_t \right\} + \tag{63}$$

$$+ \sum_0^\infty \beta^t \mu_t^2 \left\{ \frac{\gamma(1-\alpha)}{\left(1 - \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right)} \hat{A}_t - \frac{(1-\alpha)\tau}{\left(1 - \tau \frac{\overline{d}}{\overline{\overline{Y}}}\right)} \hat{LS}_t - (1 - \lambda + \lambda \tau \frac{\overline{d}}{\overline{\overline{Y}}}) \left( \hat{C}_{rt} - \hat{C}_{kt} \right) \right\} + \tag{64}$$

$$+ \sum_0^\infty \beta^t \mu_t^3 \left\{ \hat{LS}_t - \hat{w}_t - \frac{\alpha}{1-\alpha} \hat{Y}_t + \frac{1}{1-\alpha} \hat{A}_t \right\} + \tag{65}$$

$$+ \sum_0^\infty \beta^t \mu_t^4 \left\{ \hat{w}_t - \psi_w \hat{w}_{t-1} + \psi_w \hat{\Pi}_t - (1-\psi_w)(1 + \frac{\phi}{1-\alpha}) \hat{Y}_t + (1-\psi_w) \frac{\phi}{1-\alpha} \hat{A}_t \right\} \tag{66}$$

## A.6 Optimal Price Level Targeting

The solution to the Lagrangian as above boils down to the following condition.

$$\begin{aligned} \hat{\Pi}_t = & -\Theta_{LS} \left[ \left( \hat{LS}_t - \hat{LS}_t^* \right) - \left( \hat{LS}_{t-1} - \hat{LS}_{t-1}^* \right) \right] + \Theta_{\Delta_c} \left[ \left( \hat{C}_{rt} - \hat{C}_{kt} \right) - \left( \hat{C}_{rt-1} - \hat{C}_{kt-1} \right) \right] + \\ & + (\Theta_1 - \Theta_2) \left( \hat{Y}_{t-1} - \hat{A}_{t-1} \right) - \Theta_1 \left( \hat{Y}_t - \hat{A}_t \right) + \Theta_2 \left( 1 - \Theta_3 - \frac{\Theta_4}{\Theta_2} \right) \sum_{s=0}^\infty \Theta_3^s \left( \hat{Y}_{t+s} - \hat{A}_{t+s} \right) \end{aligned} \tag{67}$$

Given $\hat{\Pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$, we have:

$$\begin{aligned} \hat{P}_t = & \hat{P}_{t-1} - \Theta_{LS} \left[ \left( \hat{LS}_t - \hat{LS}_t^* \right) - \left( \hat{LS}_{t-1} - \hat{LS}_{t-1}^* \right) \right] + \Theta_{\Delta_c} \left[ \left( \hat{C}_{rt} - \hat{C}_{kt} \right) - \left( \hat{C}_{rt-1} - \hat{C}_{kt-1} \right) \right] + \\ & - (\Theta_1 - \Theta_2) \left[ \left( \hat{Y}_t - \hat{A}_t \right) - \left( \hat{Y}_{t-1} - \hat{A}_{t-1} \right) \right] + \Theta_2 \left( \hat{Y}_t - \hat{A}_t \right) - \Theta_2 \left( 1 - \Theta_3 - \frac{\Theta_4}{\Theta_2} \right) \sum_{s=0}^\infty \Theta_3^s \left( \hat{Y}_{t+s} - \hat{A}_{t+s} \right) \end{aligned} \tag{68}$$

Consider the above condition at $t = 0$ and $t = 1$ recalling that we started from the steady state.

$$\begin{aligned} \hat{P}_0 = & -\Theta_{LS} \left( \hat{LS}_0 - \hat{LS}_0^* \right) + \Theta_{\Delta_c} \left( \hat{C}_{r0} - \hat{C}_{k0} \right) - (\Theta_1 - \Theta_2) \left( \hat{Y}_0 - \hat{A}_0 \right) + \Theta_2 \left( \hat{Y}_0 - \hat{A}_0 \right) + \\ & - \Theta_2 \left( 1 - \Theta_3 - \frac{\Theta_4}{\Theta_2} \right) \sum_{s=0}^\infty \Theta_3^s \left( \hat{Y}_s - \hat{A}_s \right) \end{aligned} \tag{69}$$

$$\begin{aligned} \hat{P}_1 = & -\Theta_{LS} \left( \hat{LS}_1 - \hat{LS}_1^* \right) + \Theta_{\Delta_c} \left( \hat{C}_{r1} - \hat{C}_{k1} \right) - (\Theta_1 - \Theta_2) \left( \hat{Y}_1 - \hat{A}_1 \right) + \Theta_2 \left[ \left( \hat{Y}_1 - \hat{A}_1 \right) + \left( \hat{Y}_0 - \hat{A}_0 \right) \right] + \\ & - \Theta_2 \left( 1 - \Theta_3 - \frac{\Theta_4}{\Theta_2} \right) \left[ \sum_{s=0}^\infty \Theta_3^s \left( \hat{Y}_s - \hat{A}_s \right) + \sum_{s=0}^\infty \Theta_3^s \left( \hat{Y}_{1+s} - \hat{A}_{1+s} \right) \right] \end{aligned} \tag{70}$$

Further substituting the above conditions, we obtain the following price level targeting rule.

$$\hat{P}_t = -\Theta_{LS}\left(\hat{LS}_t - \hat{LS}_t^*\right) + \Theta_{\Delta_c}\left(\hat{C}_{rt} - \hat{C}_{kt}\right) - (\Theta_1 - \Theta_2)\left(\hat{Y}_t - \hat{A}_t\right) + \Theta_2 \sum_{j=0}^{t}\left(\hat{Y}_j - \hat{A}_j\right) +$$

$$- \Theta_2\left(1 - \Theta_3 - \frac{\Theta_4}{\Theta_2}\right)\left[\sum_{j=0}^{t}\sum_{s=0}^{\infty}\Theta_3^s\left(\hat{Y}_{j+s} - \hat{A}_{j+s}\right)\right] \quad (71)$$

## A.7 Details in Deriving RANK-Optimal Monetary Policy

The Lagrangian of the maximization problem for the RANK-Optimal policy can be written as follows.

$$L = \sum_0^\infty \beta^t \left\{-\frac{1}{2}\psi_p \hat{\Pi}_t^2 - \frac{1}{2}\frac{1+\phi}{1-\alpha}\left(\hat{Y}_t - \hat{A}_t\right)^2\right\} + \quad (72)$$

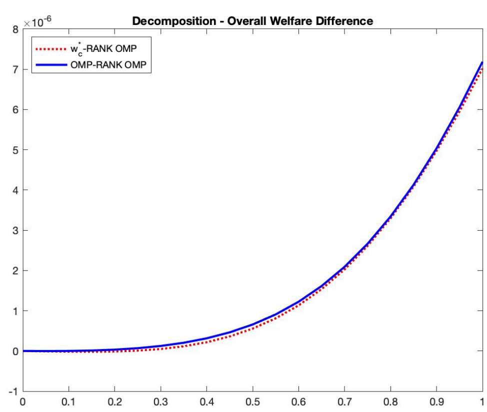
$$+ \sum_0^\infty \beta^t \mu_t^1 \left\{\hat{\Pi}_t - \beta\hat{\Pi}_{t+1} - \frac{\theta_p}{\psi_p}\hat{LS}_t\right\} + \quad (73)$$

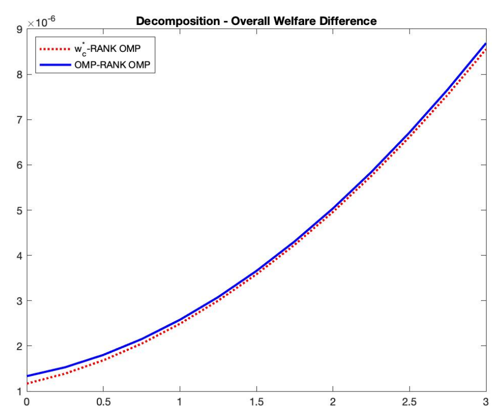
$$+ \sum_0^\infty \beta^t \mu_t^3 \left\{\hat{LS}_t - \hat{w}_t - \frac{\alpha}{1-\alpha}\hat{Y}_t + \frac{1}{1-\alpha}\hat{A}_t\right\} + \quad (74)$$

$$+ \sum_0^\infty \beta^t \mu_t^4 \left\{\hat{w}_t - \psi_w \hat{w}_{t-1} + \psi_w \hat{\Pi}_t - (1-\psi_w)(1+\frac{\phi}{1-\alpha})\hat{Y}_t + (1-\psi_w)\frac{\phi}{1-\alpha}\hat{A}_t\right\} \quad (75)$$

# B Additional Figures and Tables

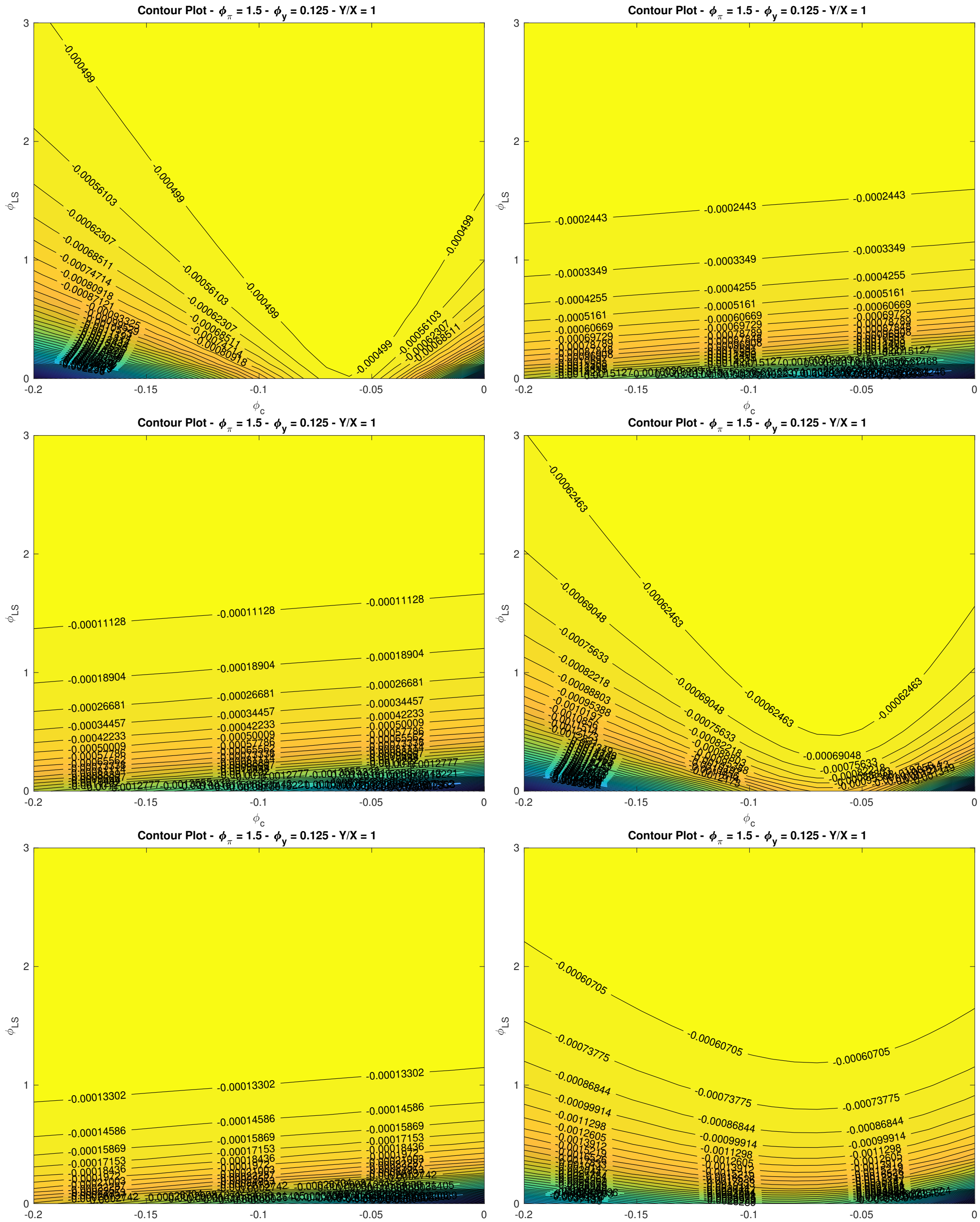
Figure B1: The importance of a non-zero weight on consumption inequality



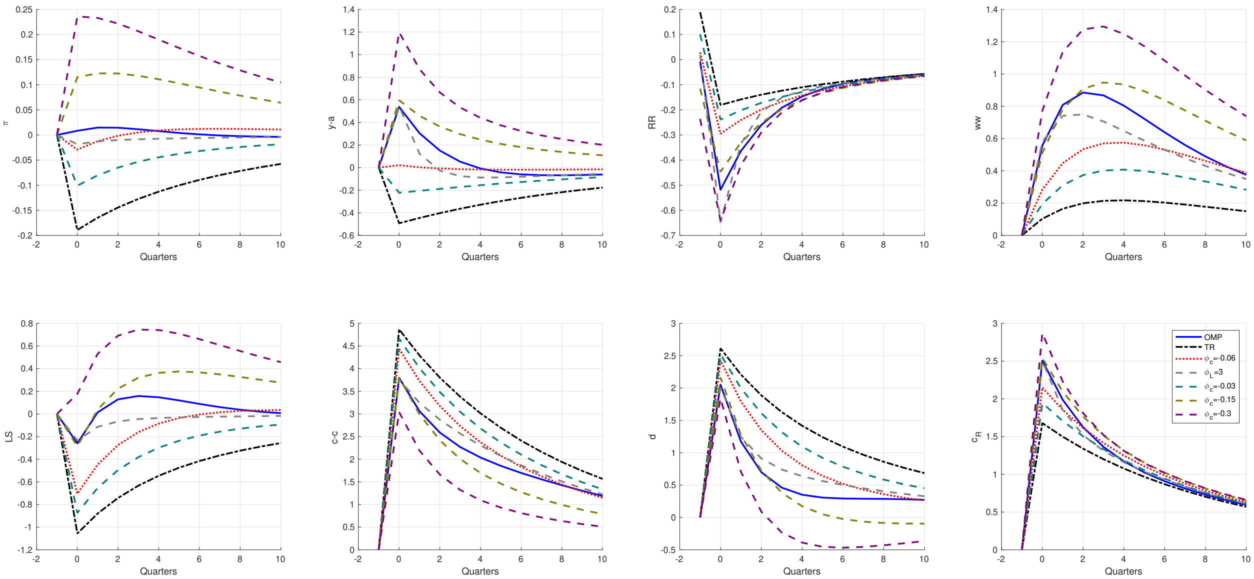

*Notes:* The figures report the consumption equivalent welfare differences between RANK Optimal Policy (RANK OMP) and two monetary policies: 1) Optimal Policy (OMP); 2) RANK Optimal Policy with a welfare weight on inequality forced to be the one of Optimal Policy $W_c^*$. The left (right) panel show the welfare differences for varying values of $\tau$ ($\gamma$). The blue line reports the difference between 1) and RANK OMP, the red line the difference between 2) and RANK OMP.

Figure B2: Welfare Contour - Shocks on/off



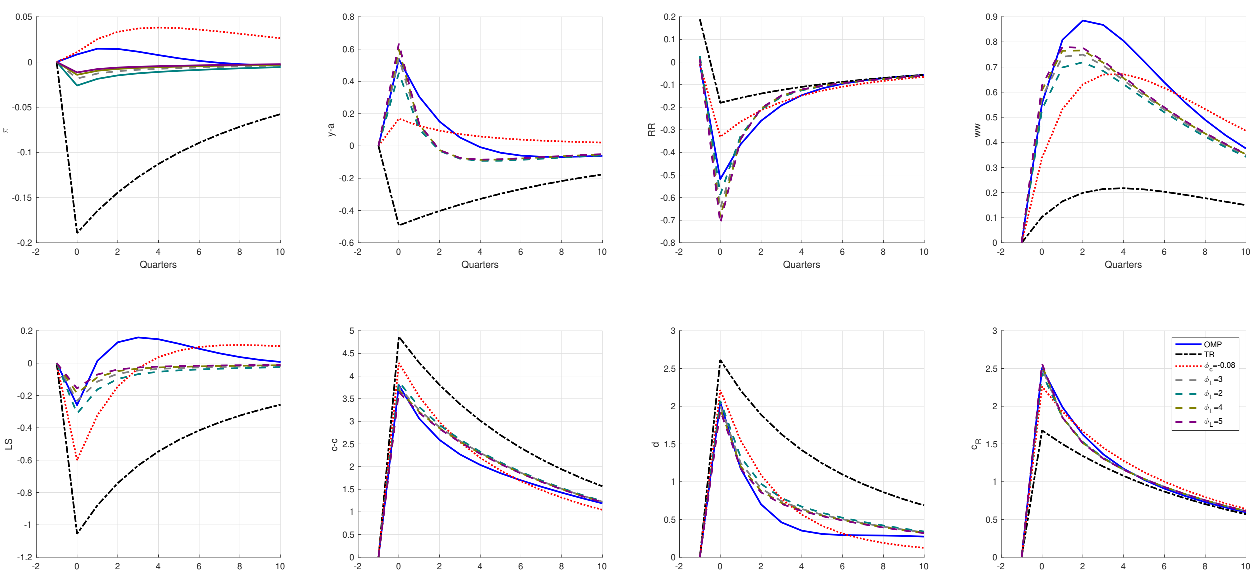

*Notes:* The figures report the consumption equivalent welfare loss, for different values of $\phi_c$ and $\phi_{LS}$. On the left column there is only 1 shock at the time. On the right column there are 2 shocks at the time. The first row corresponds to the tech shock: left is on, right is off. Similar reasoning applies to row 2 (time preference) and 3 (cost push).

Figure B3: IRF - Taylor rules with different reaction parameters




*Notes:* The figures report the impulse response functions for the augmented Taylor rule, for different values of $\phi_c$.

Figure B4: IRF - Taylor rules with different reaction parameters




*Notes:* The figures report the impulse response functions for the augmented Taylor rule, for different values of $\phi_{LS}$.

# C Adding Shocks Beyond TFP

In Section 4 we extended the model to allow for demand shocks, in the form of a time preference shock, and cost push shocks, in the form of a labor disutility shock. Specifically, instead of $\beta^t$ and $\chi$ we have $\beta_t = \beta^t(1 + \xi_t)$ and $\chi_t = \chi(1 + u_t)$, where $\xi_t$ and $u_t$ are exogenous AR(1) disturbances where relevant. We take the persistence of those shocks and the standard deviations of all three shocks from Galí and Rabanal, 2004 (0.90, 0.93, and 0.91 respectively for the tech shock, preference shock, and cost push shock) and assume that shocks are independent.[47]

The addition of other shocks in the model can introduce additional terms in the welfare criterion.[48] For simplicity, when evaluating welfare under all three shocks, we use the same welfare criterion as in Equation (26).

The table below reports the calibration's ability to match the moments identified in Galí, 2010 for our baseline case of independent shocks and for when we allow shocks to be correlated.

Table C1: Calibration of Shocks

| Target | Value from Galí, 2010 | Independent Shocks | Correlated Shocks |
|---|---|---|---|
| $\frac{\sigma(\pi)}{\sigma(y)}$ | 0.19 | 0.24 | 0.26 |
| $\frac{\sigma(L)}{\sigma(y)}$ | 0.60 | 1.31 | 1.50 |
| $\frac{\sigma(w)}{\sigma(y)}$ | 0.44 | 0.89 | 0.80 |
| $\rho(\pi, y)$ | 0.27 | 0.25 | 0.29 |
| $\rho(\pi, y)$ | 0.83 | 0.66 | 0.67 |
| $\rho(w, y)$ | 0.07 | 0.75 | 0.73 |

[47] As a robustness check, we also allow for cross correlation across the shocks. In this case, we calibrate the correlations by minimizing the sum of square deviations between the model based and empirical moments described in Table 1 in Galí, 2010. Our results are broadly unchanged.

[48] In particular, in the case of cost push shocks but not for demand shocks. Specifically, for the former there is a direct interaction between the cost push shock and the labor share, which implies that the definition of the welfare-relevant labor share changes.